Linear and Planar Array Pattern Nulling via Compressed Sensing

Jafar Ramadhan Mohammed\textsuperscript{1}, Raad H. Thaher\textsuperscript{2}, and Ahmed Jameel Abdulqader\textsuperscript{1,2},

\textsuperscript{1} College of Electronic Engineering, Ninevah University, Mosul, Iraq
\textsuperscript{2} College of Engineering, Mustansiriyah University, Baghdad, Iraq

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Abstract—An optimization method based on compressed sensing is proposed for uniformly excited linear or planar antenna arrays to perturb excitation of the minimum number of array elements in such a way that the required number of nulls is obtained. First, the spares theory is relied upon to formulate the problem and then the convex optimization approach is adopted to find the optimum solution. The optimization process is further developed by using iterative re-weighted $l_1$-norm minimization, helping select the least number of the sparse elements and impose the required constraints on the array radiation pattern. Furthermore, the nulls generated are wide enough to cancel a whole specific sidelobe. Simulation results demonstrate the effectiveness of the proposed method and the required nulls are placed with a minimum number of perturbed elements. Thus, in practical implementations of the proposed method, a highly limited number of attenuators and phase shifters is required compared to other, conventional methods.

Keywords—compressed sensing, convex optimization, iterative re-weighted $l_1$-norm minimization, linear and planar arrays.

1. Introduction

One of the challenges in current and future wireless communication systems is the presence of interfering signals that may originate either from pre-specified and known or from unknown directions. In such cases, performance of the system may be significantly degraded. This problem becomes more significant in such applications as satellites\textsuperscript{[1]}, fifth-generation wireless communications\textsuperscript{[2]} and modern radars\textsuperscript{[3]}, as the system of this type are usually expected to operate in environments characterized by severe interference and a very crowded spectrum. One of the simplest and most powerful techniques for eliminating these interfering signals is to point the nulls of the array radiation pattern in the direction of the unwanted interfering signals.

Null placement may be achieved by accurately controlling such array design variables such as element excitation weights and element spacing\textsuperscript{[4],[5]}. Conventionally, all the weights and/or positions of the array elements were perturbed to place the required nulls. Thus, the final phased array systems were usually complex, slow in their convergences, and expensive\textsuperscript{[6]}. Many researchers have investigated the complexity of such fully perturbed antenna arrays, coming up with some solutions. Some of them suggest simple deterministic approaches, such as iterative Fourier transform method\textsuperscript{[7]} and the edge-element method\textsuperscript{[8],[9]} to identify those elements that need to be perturbed for achieving the required null placement. Other scientists, meanwhile, used numerical optimization algorithms, such as the genetic algorithm\textsuperscript{[4],[10]}, particle swarm optimization\textsuperscript{[11]}, simulated annealing\textsuperscript{[12]}, evolutionary algorithms\textsuperscript{[13]}, adaptive algorithms\textsuperscript{[14]}, cuckoo search optimization\textsuperscript{[15]}, invasive weeds optimization\textsuperscript{[16]}, and grey wolf optimization\textsuperscript{[17]}, to optimize the excitations of the perturbed elements. None of the aforementioned techniques offers a clear path towards selecting the minimum required number of perturbed elements needed in order to place the required number of nulls. In fact, they always assumed that the number of the perturbed elements should be higher than the total number of the required nulls in order to insure an accurate pattern nulling capability. Thus, the number of the perturbed elements was excessive and the solutions were usually not optimal.

Other methods include the use of clustered arrays in which the main arrays were divided into clusters that may consist of either regular or irregular clustered elements\textsuperscript{[18]–[20]}. Furthermore, paper\textsuperscript{[21]} suggested a partially thinned array approach that was applied to side elements only, thus creating a relatively low complexity null placement method. In addition, the structure of a conventional adaptive sidelobe canceller system used in spaced radars has been greatly simplified by using different auxiliary configurations\textsuperscript{[22]} in order to create another solution to this important issue.

In light of the above discussions, there is a great need for a new optimized method that is capable of perturbing only the exactly required number of elements in order to place the required number of nulls in an efficient manner. In a bid to solve the problem, compressed sensing was suggested in\textsuperscript{[23],[24]} in order to significantly reduce complexity of array feeding networks. In\textsuperscript{[25]}, Bayesian compressed sensing was suggested to find the best match between the sparse array and the reference patterns. Generally, several sparse recovery algorithms exist, such as Yalli\textsuperscript{[26]}, smoothed $l_0$-norm\textsuperscript{[27]}, orthogonal matching pursuit\textsuperscript{[28]}, and iterative hard threshold\textsuperscript{[29]} that may be used to solve...
the complexity problem and achieve the desired patterns. Some of these algorithms, like Yali and smoothed $l_0$-norm, usually do not accurately recover the sparse solutions. On the other hand, the iterative reweighted $l_1$-norm [30] and the two-steps $l_0$ [31] methods were used to efficiently determine the minimum number of perturbed elements and to achieve the desired constraints.

In this paper, radiation patterns of linear and planar arrays are optimized by means of the compressed sensing approach, making sure that the required nulls are placed under a minimum number of perturbed elements. First, the sparse recovery array is built, and then it is implemented with convex optimization applied in order to find the optimum solution. The sparsity of the solution is enhanced through the use of the iterative reweighted $l_1$-norm algorithm [32]. That approach allowed the required nulls to be placed efficiently with precisely the needed number of perturbed elements.

2. Principles of the Method

For simplicity, consider a linear array of $N$ isotropic elements in which the array pattern may be expressed by:

$$AF_{\text{uniform}}(\theta) = \sum_{n=1}^{N} A_{\text{on}} e^{jkd_n u_n},$$

where $\theta$ is the observation angle around the array axis, $k = \frac{2\pi}{\lambda}$ is the wave number, $\lambda$ is the wavelength, $u = \sin \theta$, $d_n$ is the position of elements along the $x$ axis which is represented by $d_n = (n - \frac{N}{2})d$. and $d$ is the element spacing. Further, $x_{\text{on}} = A_{\text{on}} e^{-jkd_n u_n}$, where $A_{\text{on}}$ is the array amplitude, $u_n = \sin \theta_n$, and $\theta_n$ is the steering angle of the main beam.

After substitution, Eq. (1) can be rewritten as:

$$AF_{\text{uniform}}(\theta) = \sum_{n=1}^{N} x_{\text{on}} e^{jkd_n (u - u_n)}.$$ (2)

To place a number of wide nulls equal to $Q$, where $q = 1, 2, \ldots, Q$, we need to perturb the element weights as follows:

$$X_n = x_{\text{on}} + x_n,$$ (3)

where $x_n$ is the weight of the sparse elements. The array factor at the null directions is equal to zero, $AF(\theta_q) = 0$. Then Eq. (2) can be modified accordingly:

$$AF(\theta_q) = \sum_{n=1}^{N} X_n e^{jkd_n u_n} = \sum_{n=1}^{N} (x_{\text{on}} + x_n) e^{jkd_n u_n},$$ (4)

which can be rewritten as:

$$AF_{\text{uniform}}(\theta_q) = -\sum_{n=1}^{N} x_n e^{jkd_n u_n}.$$ (5)

This is a set of linear equations that can be written as $Ax = b$ where $A = \sum_{n=1}^{N} e^{jkd_n u_n}$, $x = x_n$, and $b = -AF_{\text{uniform}}(\theta_q)$. Note that vector $x$ with size $N \times 1$ is the sparse weight that needs to be found and it contains both zero and non-zero values. Vector $b$ with size $Q \times 1$ is the magnitude of the uniform array pattern at null directions with the opposite phase and, finally, $A$ is the matrix with size $Q \times N$. These three parameters can be written as:

$$x = [x_1, x_2, \ldots, x_N]^T,$$ (6)

$$b = [-AF_{\text{uniform}}(\theta_1), -AF_{\text{uniform}}(\theta_2), \ldots, -AF_{\text{uniform}}(\theta_Q)]^T,$$ (7)

$$A = \begin{bmatrix} e^{-jkd_1 u_1} & \cdots & e^{-jkd_N u_1} \\ \vdots & \ddots & \vdots \\ e^{-jkd_1 u_Q} & \cdots & e^{-jkd_N u_Q} \end{bmatrix}.$$ (8)

Suppose that the number of array elements is larger than the number of the required nulls, $Q < N$, which is the practical scenario, especially for large arrays that consist of hundreds of elements. In such a case, the system will have infinite solutions. To find the minimum number of the perturbed elements, Eq. (8) can be solved using the $l_0$ norm minimization approach. However, the problem becomes non-convex and cannot be solved by means of convex optimization. Thus, it is first converted to the convex type by using the $l_1$ norm that can be solved by:

$$\text{minimize } ||x||_1 \text{ subject to } Ax - b \leq \varepsilon.$$ (9)

This equation can be solved by using the convex optimization approach, with its implementation explained in [33]. To enhance the sparsity of the solutions and hence reduce the number of perturbed elements, Eq. (9) can be iteratively minimized as:

$$\text{minimize } ||\beta(x^{(i-1)}x^t)||_1 \text{ subject to } Ax - b \leq \varepsilon,$$ (10)

where $\beta = \frac{1}{1 + \delta^2}$ and $\delta$ is a small positive number that is used to provide stability to array weights, such that the minimization process is assured by estimating non-zero values of the sparse elements and is not affected by the zero values in the next new iterations. The non-zero values of Eq. (10) represent the sparse elements that give exactly the needed number of the perturbed elements. The corresponding array pattern of these perturbed elements can be obtained. Then, the overall array pattern can be computed by:

$$AF_{\text{proposed}} = AF_{\text{uniform}} - AF_{\text{sparse elements}}.$$ (11)

3. Simulation Results

This section demonstrates the performance of the proposed method while generating the required nulls under the minimum number of perturbed elements. In all scenarios, the number of iterations, $i$, and $\delta$ are chosen to be 15 and $10^{-6}$ respectively.
3.1. Scenario 1 – Linear Array

A uniform linear array of \( N = 20 \) elements with inter-element spacing equal to \( d = 0.5\lambda \) is considered in this scenario. Several cases of null placements are investigated to illustrate the effectiveness of the proposed method. In the first case, a single wide null centered at \( 49^\circ \) is placed.

To obtain such a wide null, two adjacent sharp nulls have to be imposed. For example, to place a wide null centered at \( 49^\circ \) two adjacent sharp nulls are placed at \( 48^\circ \) and \( 50^\circ \), respectively. Each sharp null needs one perturbed element and, thus, each wide null will need at least two perturbed elements. Further, to achieve such a wide null, it is required that the patterns of the spare array and the uniform array according to Eq. (11) are exactly coincident at the null direction of \( 49^\circ \).

Figure 1 shows the results of the proposed array using the compressed sensing approach. The uniform array pattern is also shown for comparison. It can be seen that the required wide null has been successfully placed with only two sparse (non-zero) elements. The perturbed complex weights of these two sparse elements selected randomly by the algorithm are \( 0.5176 + j0.2652 \) and \( 0.5176 - j0.2652 \), with indices 1 and 20, respectively.

In the second case, multiple wide nulls are generated. Accordingly, the number of the randomly perturbed elements is expected to increase. However, this increment represents the actual need of the algorithm to perform the required null placement. To highlight this important point, the relationship between the required number of nulls and the minimum number of perturbed elements is illustrated in Figure 2.

### Table 1

<table>
<thead>
<tr>
<th>Indices of sparse elements</th>
<th>Complex values of sparse elements</th>
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<tbody>
<tr>
<td>1</td>
<td>0.5208 + j0.3099</td>
</tr>
<tr>
<td>4</td>
<td>-0.2612 + j0.0487</td>
</tr>
<tr>
<td>17</td>
<td>-0.2612 - j0.0487</td>
</tr>
<tr>
<td>20</td>
<td>0.5208 - j0.3099</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Design parameters of two and three wide nulls</th>
</tr>
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<tbody>
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<td>( A )</td>
</tr>
<tr>
<td>4 \times 20 matrix</td>
</tr>
<tr>
<td>6 \times 20 matrix</td>
</tr>
</tbody>
</table>

Fig. 1. Radiation patterns of the tested arrays (a) and the corresponding perturbed elements of the proposed array for \( N = 20 \) and a single wide null at \( 49^\circ \) (b). (See digital edition to find the color version).

Fig. 2. The minimum number of perturbed elements versus the required number of wide nulls.
Fig. 3. Radiation patterns of the tested arrays (a) and the corresponding perturbed elements of the proposed array for \(N = 20\) and two wide nulls at 30° and 60° (b).

Fig. 4. Radiation patterns of the tested arrays (a) and the corresponding perturbed elements of the proposed array for \(N = 20\) and three wide nulls at 20°, 45°, and 55°.

Fig. 5. Radiation patterns of the proposed planar array (a) and the uniform array pattern (b) for \(20 \times 20\) and a single wide null centered at \(v = -0.5\).

Figures 3 and 4 show the results of the tested arrays of two and three wide nulls, respectively. These results fully confirm the effectiveness of the proposed array in placing the required number of wide nulls. Moreover, the directivity of the proposed array is found to be affected only slightly, as long as the number of perturbed elements is lower than the total number of array elements.

3.2. Scenario 2 – Planar Array

In this scenario, a square planar array with \(20 \times 20\) elements and \(\lambda/2\) inter-element spacing along the x and y axes was considered. In the first use case, the center of the required wide null was chosen to be at \(v = -0.5\) and no nulls at \(u\) plane were presented, with

\[
\begin{align*}
    v &= \sin(\theta) \sin(\phi), \\
    u &= \sin(\theta) \cos(\phi).
\end{align*}
\]

Figure 5a shows the three-dimensional pattern of the proposed array obtained with the use of the compressed sensing approach, characterized by the minimum number of perturbed elements, while Fig. 5b shows the results of the original uniform planar array shown for comparison purposes.

In the other use case, two wide nulls centered at \(v = -0.5\) and \(u = -0.7\) are considered. Figure 6 shows the results of this case, with the two required nulls placed successfully.
An efficient and simple optimization method based on the sparse theory and on the compressed sensing approach was presented for synthesizing linear and planar array patterns with the minimum number of perturbed elements. The simplicity of the proposed method means that the number of the RF components, such as attenuators and phase shifters, is reduced. The proposed array is capable of placing the required wide nulls at undesired directions. For each single wide null there is a need for at least two perturbed elements.

The results show a significant reduction in the number of sparse elements needed to perform the desired null placements. For example, with the minimum number of perturbed elements, the proposed array is capable of placing the corresponding perturbed amplitudes and phases.

4. Conclusions

Convex programming has been applied to implement and find sparse elements needed to perform the desired null placements. The results show a significant reduction in the complexity of the array feeding network.

References


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Jafar Ramadhan Mohammed received the B.Sc. and M.Sc. degrees in Electronics and Communication Engineering in 1998, and 2001, respectively, and the Ph.D. degree in Digital Communication Engineering from Panjab University, India in 2009. He was a Visiting Lecturer in the Faculty of Electronics and Computer Engineering at the Malaysia Technical University Melaka (UTeM), Melaka, Malaysia in 2011 and Autonoma University of Madrid, Spain in 2013. He is currently a Professor and Vice Chancellor for Scientific Affairs at Ninevah University. His main research interests are in the area of digital signal processing and its applications, antenna, and adaptive arrays.

Raad Hamdan Thaheer received the M.Sc. degree in Electronics and Communication Engineering in 1981, and the Ph.D. degree in Communication Engineering from Faculty Polytechnic University Bucharest Romania in 1997. He works as a professor in Mustansiriyah University, College of Engineering, Electrical Engineering Department, Baghdad, Iraq. His specialization is in the electronic communication engineering and his research interests are in the field of communication systems, electronics, microwaves, antennas, and communication networks.

Ahmed Jameel Abdulqader received the B.Sc. and M.Sc. degrees in Electronics and Communication Engineering from the University of Mosul, Iraq, in 2009, and 2013, respectively. He is currently a Ph.D. student at Mustansiriyah University, Baghdad, Iraq. He is Lecturer at Ninevah University. His main research interests are in the area of design and analysis of antenna arrays, array pattern optimization, mobile communication systems, and computer networks.

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