Analysis of Selective-Decode and Forward Relaying Protocol over κ-μ Fading Channel Distribution

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Abstract—In this work, the performance of selective-decode and forward (S-DF) relay systems over κ-μ fading channel conditions is examined in terms of probability density function (PDF), system model and cumulative distribution function (CDF) of the κ-μ distributed envelope, signal-to-noise ratio and the techniques used to generate samples that rely on κ-μ distribution. Specifically, we consider a case where the source-to-relay, relay-to-destination and source-to-destination link is subject to independent and identically distributed κ-μ fading. From the simulation results, the enhancement in the symbol error rate (SER) with a stronger line of sight (LOS) component is observed. This shows that S-DF relaying systems may perform well even in non-fading or LOS conditions. Monte Carlo simulations are conducted for various fading parameter values and the outcomes turn out to be a close match for theoretical results, which validates the derivations made.

Keywords—channel fading, channel state information, relaying protocol, selective decode and forward, symbol error rate.

1. Introduction

Fifth generation (5G) wireless communication systems will require a paradigm shift to meet the increasing demand for reliable connectivity offering high data rates, low latency, better energy efficiency, and femto cell-based relays [1]–[3]. Cooperative communication is the natural choice for 5G wireless communication systems, and is adopted in third generation partnership project (3GPP), universal mobile telecommunications service (UMTS), long term evolution (LTE)-Advanced and IEEE 802.11 standards, because the nodes of a cooperative communication network may share their resources with each other while transmitting the signal [4]–[6]. This approach is also incorporated into numerous 5G wireless applications, such as machine-to-machine (M2M), device-to-device (D2D), cognitive radio (CR), high speed terrestrial network (HSTN), and free space optical (FSO) communication [7]–[10].

Relay-assisted cooperation is the first step towards a 5G system that is expected to deliver up to 20 Gbps in downlink (DL) and 10 Gbps in uplink (UL) communications, serving as a benchmark for network operators during initial rollouts over the next few years [11]–[12]. The relay infrastructure does not require a wired network connection, thus offering a reduction in the operator’s backhaul costs. Through the additional cooperative diversity inherent in such wireless systems, cooperative wireless communication significantly improves end-to-end reliability. If the direct source-to-destination (SD) channel is in a deep fade, the main advantage of the cooperative communication is that the destination node may still receive the source signal via the relay node.

Two basic relaying methods used for transmitting and receiving signals may be distinguished: analog and digital. Analog relaying is also referred to as non-regenerative, as the signals are not required to be digitized before they are sent. Amplify-and-forward (AF) is an example of the analog relaying approach. On the other hand, before transmitting the signals to their destination, a relay node uses the digital relay protocol to decode and encode signals. Consequently, digital relaying is also known as regenerative relaying. The main drawback of the AF protocol is noise amplification. The relay may transmit the erroneous signal to the destination node if the decode-and-forward (DF) relaying protocol is used. The S-DF protocol has been proposed to overcome the disadvantage of noise amplification and erroneous decoding related to AF and DF, respectively [13]–[17]. To overcome the problem of noise amplification and relay error propagation, the S-DF protocol is used in 5G wireless systems. S-DF relaying networks relay forward correctly decoded signals only. Otherwise, they remain idle. Moreover, network connectivity and data transmission rates of S-DF relaying may be further augmented by using multiple-input multiple-output (MIMO) in conjunction with space-time-block-code (STBC) [18].

In [19]–[20], the authors investigated an S-DF relaying network over the Rayleigh flat fading channel. In [19], the authors investigated pairwise error probability (PEP) performance of an S-DF relaying network over the Rayleigh flat fading channel, evaluating ideal channel conditions. In [21]–[23], the authors investigated an S-DF relaying network over Nakagami-m fading channel conditions. In [23], the authors investigated a dual hop (DH) S-DF
relying network over frequency flat Nakagami-\(m\) fading channel conditions, evaluating ideal channel conditions. However, papers [19]–[23] did taken non-homogeneous fading channel conditions into consideration. The performance of wireless communication systems is significantly influenced by stochastic modeling and characterization of the fading links between the communicating nodes. Therefore, accurate stochastic modeling is very critical in the development of efficient wireless communication schemes and protocols.

In the literature, a variety of stochastic/statistical distributions have been developed to model small-scale fluctuations in the transmitted signal envelope over fading channels, such as Nakagami-\(m\), Rayleigh and Weibull [24]–[26]. However, none of the stochastic models described capture the non-linearity of the propagation medium. \(\kappa\)-\(\mu\) distribution is a suitable choice for LOS applications and, on the other hand, for non-LOS, \(\eta\)-\(\mu\) fading distribution is better suited for non-LOS scenarios. In [27], the authors proposed \(\eta\)-\(\mu\) and \(\kappa\)-\(\mu\) non-homogeneous fading distributions for LOS and non-LOS components, respectively. In [28]–[33], the authors investigated a relaying network under \(\kappa\)-\(\mu\) and \(\eta\)-\(\mu\) fading channel conditions. In [34], the authors investigated the SER performance of a DF relaying network over \(\eta\)-\(\mu\) and \(\kappa\)-\(\mu\) fading channel conditions. The exact SER expression is derived for M-ary phase shift keying (PSK) modulated schemes. In this work, SER expressions are obtained for S-DF relaying networks and additional diversity gains are achieved due to use of MIMO in conjunction with STBC. Hence, we consider \(\kappa\)-\(\mu\) fading distribution to be well suited for LOS applications.

The paper is organized as follows. In Section 2, SER is investigated over \(\kappa\)-\(\mu\) fading channel conditions. The closed form SER expression is derived using a moment generating function (MGF)-based approach. In Section 3, simulation results are given and Section 4 presents the conclusions.

2. End-to-end Symbol Error Analysis

Let us consider a MIMO-STBC S-DF relaying network with a \(K\) number of relay nodes. In all the analyses associated with SER performance of S-DF relaying networks, we will assume \(N_s \times N_r\) MIMO systems. \(N_s\) is the number of antennas existing at the source node and \(N_r\) is the number of antennas installed at the relay nodes. Since both source and relay nodes use the same orthogonal STBC code, we take \(N_s = N_r = N\). It is presumed that the orthogonal STBC code is conveyed over \(T\) time slots. An orthogonal STBC codeword for complete STBC communication may be agreed by a matrix with dimensions \(N_s \times T\).

It was argued before that orthogonal STBC codewords may be managed in different aerials and then the data processed may be combined together in order to obtain effective results - an approach that is similar to maximum ratio combiner (MRC) [35]. Orthogonal STBC is designed such that the vectors representing any pair of columns taken from the source-to-\(r\)-th relay coding matrix \(H_{sr}\) are orthogonal, i.e. STBC converts the vector channel into a scalar channel. For orthogonal STBC designs, conditional SNR at the receiver may be given as an Euclidean or a Frobenius norm of the channel times the average SNR, as [4]:

\[
g_{sr} = \gamma_{sr} \|H_{sr}\|_F^2, \quad (1)
\]

where \(\gamma_{sr}\) denotes the conditional SNR of the source-to-\(r\)-th relay fading link, \(\gamma_{sr}\) denotes the average conditional SNR of the source-to-\(r\)-th relay fading link and \(\|H_{sr}\|_F^2\) denotes the Frobenius norm or \(L_2\) norm. All elements of \(H_{sr}\) are i.i.d. \(\kappa\)-\(\mu\) distributed random variables (RVs). The suggestion of \(\kappa\)-\(\mu\) channel fading was given in [36] as a generalized distribution to model a non-homogeneous fading environment. As for \(\eta\)-\(\mu\) and Nakagami-\(m\) fading channel distribution, it was observed that, for \(\kappa\)-\(\mu\) fading channel conditions, the multipath components form clusters. Each cluster has several scattered multipath components. The delay spread of different clusters is relatively larger than the delay spread of multipath components within a cluster. Every cluster is assumed to have the same average power. Unlike in \(\eta\)-\(\mu\) fading and like Nakagami-\(m\) fading, it is assumed that the in-phase and quadrature phase components are independent and have equal powers in \(\kappa\)-\(\mu\) fading. However, each cluster is assumed to have some dominant components considered to be of the LOS variety. In such a model, the representation of the envelope of the fading signal is slightly different from that of Nakagami-\(m\) and/or \(\eta\)-\(\mu\) fading. It may be given as [36]–[39]:

\[
\beta^2 = \sum_{j=0}^{J} \left( (I_j + \phi_j)^2 + (\Omega_j + \alpha_j)^2 \right) , \quad (2)
\]

where \(J\) is the number of clusters in the received signal, \((I_j + \phi_j)\) and \((\Omega_j + \alpha_j)\) are the in-phase and quadrature phase components, respectively, of the resultant signal of

<table>
<thead>
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<th>Table 1</th>
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<td>Modulation parameters for various modulation schemes [34]–[35]</td>
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<tr>
<td>Modulation scheme</td>
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</tr>
<tr>
<td>Binary PSK</td>
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<tr>
<td>Binary frequency shift keying</td>
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<tr>
<td>M-ary-PSK</td>
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<td>M-ary-pulse amplitude modulation</td>
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<td>M-ary-quadrature amplitude modulation (QAM)</td>
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where \( I_j \) and \( \Omega_j \) are mutually independent and zero-mean circular-shift complex Gaussian (ZMCS CG), \( E[I_j] = E[\Omega_j] = 0 \) and have equal variance, i.e. \( E[I_j^2] = E[\Omega_j^2] = \sigma^2 \). \( \phi_j \) and \( \alpha_j \) denote the in-phase and quadrature phase components, respectively. The non-zero mean of in-phase and quadrature phase components reveals the presence of a dominant component in the clusters of the received signal. Again, as in the case of Nakagami-\( m \) and \( \eta-\mu \) fading models, the fading amplitude may be expressed as:

\[
\beta^2 = \sum_{j=0}^{j} C_j^2
\]

(3)

\[
C_j^2 = (I_j + \phi_j)^2 + (\Omega_j + \alpha_j)^2.
\]

(4)

From the fact that \( I_j \) and \( \Omega_j \) are Gaussian distributed, it is to be noted here that \( C_j^2 \) follows non-central Chi-squared distribution. Unlike Nakagami-\( m \) and \( \eta-\mu \) channel models, \( \kappa-\mu \) distribution is suitable for model LOS environments. PDF of SNR for an STBC MIMO system over \( \kappa-\mu \) fading channels may be given by [36]-[39]:

\[
p_{\gamma_{fr}}(\gamma_{fr}) = \frac{\mu_{sr}N_sN_R(1 + \kappa_{fr})}{(\kappa_{fr})^{\frac{\mu_{sr}N_sN_R+1}{2}}e^{\mu_{sr}N_sN_R}} \frac{\mu_{sr}N_sN_R(1 + \kappa_{fr})}{(\kappa_{fr})^{\frac{\mu_{sr}N_sN_R+1}{2}}} e^{-\frac{\mu_{sr}N_sN_R(1 + \kappa_{fr})}{\gamma_{fr}}} I_{\mu_{sr}N_sN_R-1} \left( 2\mu_{sr}N_sN_R \sqrt{\frac{\kappa_{fr}(1 + \kappa_{fr})}{\gamma_{fr}}} \right),
\]

(5)

where \( \mu_{sr} > 0 \) is the channel fading parameter directly related to the number of clusters, \( \kappa_{fr} \) denotes the ratio of power in the LOS components to that of scattered components. Note that \( \gamma_{fr} \) is the expected SNR and is used as a scaling factor of \( ||H_{fr}||^2 \) in existing literature to indicate average SNR at the receiver. The instantaneous SER \( P_{E}^{R}(\gamma_{fr}) \) of the source-to-\( r \)-th relay fading link may be expressed as [40]-[41]:

\[
P_{E}^{R}(\gamma_{fr}) = aQ\left(\sqrt{b\gamma_{fr}}\right) - cQ\left(\sqrt{b\gamma_{fr}}\right),
\]

(6)

where \( a, b \) and \( c \) are modulation-dependent parameters listed in Table 1 and \( Q(.) \) represents the Gaussian \( Q \) function describing the area under the tail of a Gaussian curve and is defined as [40]-[41]:

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt = \frac{1}{\sqrt{\pi}} \int_{x}^{\infty} e^{-z^2} dz = \frac{\text{erfc}(x)}{2},
\]

(7)

where \( \text{erfc}(x) \) is the complementary error function, which is accessible, inter alia, in Matlab.

The expected SER \( P_{E}^{R} \) may be obtained by taking expectation of the instantaneous SER over the PDF of receiving instantaneous SNR. For averaging the conditional SER, we will use the MGF-based approach. It may be expressed as [40]:

\[
P_{E}^{R} = \frac{\alpha}{\pi} \int_{0}^{\frac{\pi}{2}} M_{\gamma_{fr}} \left( \frac{b}{2\sin^2 \theta} \right) d\theta - \frac{\alpha}{\pi} \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} M_{\gamma_{fr}} \left( \frac{b}{2\sin^2 \theta} \right) d\theta,
\]

(8)

where \( M_{\gamma_{fr}}(.) \) is the MGF of the received conditional SNR. MGF of \( \kappa-\mu \) distributed instantaneous SNR is given as [40]-[41]:

\[
M_{\gamma_{fr}}(s) = \int_{0}^{\infty} p_{\gamma_{fr}}(\gamma_{fr})e^{-\gamma_{fr}d\gamma_{fr}} = \left( \frac{\mu_{sr}N_sN_R(1 + \kappa_{fr})}{(2\mu_{sr}N_sN_R(1 + \kappa_{fr}) + \sqrt{\gamma_{fr}}) \sqrt{\gamma_{fr}}} \right)^{\mu_{sr}N_sN_R} e^{\left( \frac{\mu_{sr}N_sN_R(1 + \kappa_{fr})}{2\mu_{sr}N_sN_R(1 + \kappa_{fr}) + \gamma_{fr}} \right)} - \kappa_{fr} \mu_{sr}N_sN_R.
\]

(9)

\( I_1 \) may be expressed in terms of a confluent hypergeometric function [42], as evaluated in Appendix A.

\[
I_1 = \frac{a}{\pi} \sqrt{\gamma_{fr}(\mu_{sr}N_sN_R(1 + \kappa_{fr}))^{\mu_{sr}N_sN_R}} \frac{2\mu_{sr}N_sN_R(1 + \kappa_{fr})}{2\mu_{sr}N_sN_R(1 + \kappa_{fr}) + \sqrt{\gamma_{fr}}} \left( \frac{\mu_{sr}N_sN_R}{\sqrt{\gamma_{fr}}} \right)^{\mu_{sr}N_sN_R + 1} I_{\mu_{sr}N_sN_R + 1} \left( 1 \right) \Gamma \left( \frac{\mu_{sr}N_sN_R + 1}{2} \right) \beta_{\mu_{sr}N_sN_R + 1} \frac{\left( 2\mu_{sr}N_sN_R(1 + \kappa_{fr}) \right)^{\mu_{sr}N_sN_R + 1}}{\left( 2\mu_{sr}N_sN_R(1 + \kappa_{fr}) + \sqrt{\gamma_{fr}} \right)^{\mu_{sr}N_sN_R + 1}} \sqrt{\gamma_{fr}} \Gamma \left( \frac{\mu_{sr}N_sN_R + 1}{2} \right).
\]

(10)
where $\Gamma(x)$ represents the Gamma function [42] and $(x)_n$ denotes the descending factorial [44]–[45], $(x)_n = \frac{\Gamma(x+1)}{\Gamma(x-n+1)}$.

$I_2$ may be expressed in terms of confluent Lauricella’s hypergeometric function [43], as evaluated in Appendix B.

\[
I_2 = \frac{c e^{\sqrt{b_2 b_4}} (\mu_\tau N_2 N_2 N_2 \kappa_\tau)^{\mu_2 N_2 N_2}}{2 \pi \sqrt{2 \mu_\tau N_2 N_2 N_2 (1 + \kappa_\tau)}} \left( \frac{\mu_\tau N_2 N_2 (1 + \kappa_\tau)}{\mu_\tau N_2 N_2 (1 + \kappa_\tau) + b_{\tau_2}} \right)^{\mu_2 N_2 N_2 + \frac{1}{2}} \frac{\Gamma(\mu_\tau N_2 N_2 + \frac{1}{2}) \sqrt{\pi}}{\Gamma(\mu_\tau N_2 N_2 + 1)} \times \left( \frac{\mu_\tau N_2 N_2 + \frac{1}{2}}{\mu_\tau N_2 N_2 + 1} \right)^{n} \right) \right) \right)^{n}. \tag{11}
\]

Following a similar analysis, the SER for the $S \rightarrow D$ fading link may be given as [40]:

\[
P_{E-S-D} = \frac{a}{\pi} \int_{0}^{\frac{\pi}{2}} M_{\text{snr}} \left( \theta \right) d\theta - \frac{c}{\pi} \int_{0}^{\frac{\pi}{2}} M_{\text{snr}} \left( \theta \right) d\theta , \tag{12}
\]

where

\[
K_1 = \frac{a}{\pi} \sqrt{2 \pi} \sqrt{\gamma_{\text{snr}}} \left( \frac{2 \mu SD N_2 N_2 (1 + \kappa_{SD})}{2 \mu SD N_2 N_2 (1 + \kappa_{SD}) + b_{\text{snr}}^2} \right)^{\mu SD N_2 N_2 + \frac{1}{2}} \frac{\Gamma(\mu SD N_2 N_2 + \frac{1}{2}) \sqrt{\pi}}{\Gamma(\mu SD N_2 N_2 + 1)} \times \left( \frac{\mu SD N_2 N_2 + \frac{1}{2}}{\mu SD N_2 N_2 + 1} \right)^{n} \right) \right) \right)^{n}. \tag{13}
\]

where $\mu_{SD} > 0$ is the channel fading parameter directly related to the number of clusters and $\kappa_{SD}$ denotes the ratio of power in the LOS components to that of scattered components for $S \rightarrow D$ fading links. $\gamma_{\text{snr}}$ denotes the average SNR of the source-to-destination fading link.

\[
K_2 = \frac{c}{\pi} \sqrt{2 \pi} \sqrt{\gamma_{\text{snr}}} \left( \frac{2 \mu SD N_2 N_2 (1 + \kappa_{SD})}{2 \mu SD N_2 N_2 (1 + \kappa_{SD}) + b_{\text{snr}}^2} \right)^{\mu SD N_2 N_2 + \frac{1}{2}} \frac{\Gamma(\mu SD N_2 N_2 + \frac{1}{2}) \sqrt{\pi}}{\Gamma(\mu SD N_2 N_2 + 1)} \times \left( \frac{\mu SD N_2 N_2 + \frac{1}{2}}{\mu SD N_2 N_2 + 1} \right)^{n} \right) \right) \right)^{n}. \tag{14}
\]

Also, the SER for the $R \rightarrow D$ fading link may be expressed [40] as:

\[
P_{E-R-D} = \frac{a}{\pi} \int_{0}^{\frac{\pi}{2}} M_{\text{snr}} \left( \theta \right) d\theta - \frac{c}{\pi} \int_{0}^{\frac{\pi}{2}} M_{\text{snr}} \left( \theta \right) d\theta , \tag{15}
\]
where

\[
\psi_1 = \frac{a}{\pi} \frac{\sqrt{b} \theta_{rd} (\mu_{RD} N_{RD} N_{D} \kappa_{RD})^{\mu_{RD} N_{RD} N_{D}}}{\sqrt{2 \pi \mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD})}} \left( \frac{2 \mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD})}{2 \mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD}) + b \theta_{rd}} \right)^{\mu_{RD} N_{RD} N_{D} + \frac{1}{2}} \frac{\Gamma \left( \mu_{RD} N_{RD} N_{D} + \frac{1}{2} \right) \sqrt{\pi}}{\Gamma (\mu_{RD} N_{RD} N_{D} + 1)} \times \\
\sum_{J=0}^{\infty} \sum_{n=0}^{\infty} \left( \mu_{RD} N_{RD} N_{D} + \frac{1}{2} \right) \frac{(1)^{J+n+1} (2)^{J+n+1} \left( \frac{2 \mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD})}{2 \mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD}) + b \theta_{rd}} \right)^{J+n+1}}{(\mu_{RD} N_{RD} N_{D} + 1)^{J+n+1}}.
\]

(16)

\[
\psi_2 = \frac{c}{\pi} \frac{\sqrt{b} \theta_{rd} (\mu_{RD} N_{RD} N_{D} \kappa_{RD})^{\mu_{RD} N_{RD} N_{D}}}{2 \pi \sqrt{2 \mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD})}} \left( \frac{\mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD})}{\mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD}) + b \theta_{rd}} \right)^{\mu_{RD} N_{RD} N_{D} + \frac{1}{2}} \frac{\Gamma \left( \mu_{RD} N_{RD} N_{D} + \frac{1}{2} \right) \sqrt{\pi}}{\Gamma (\mu_{RD} N_{RD} N_{D} + 1)} \times \\
\sum_{J=0}^{\infty} \sum_{n=0}^{\infty} \left( \mu_{RD} N_{RD} N_{D} + \frac{1}{2} \right) \frac{(1)^{J+n+1} (2)^{J+n+1} \left( \frac{\mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD})}{\mu_{RD} N_{RD} N_{D} (1 + \kappa_{RD}) + b \theta_{rd}} \right)^{J+n+1}}{(\mu_{RD} N_{RD} N_{D} + 1)^{J+n+1}}.
\]

(17)

In the above equations \( \mu_{RD} > 0 \) is the channel fading parameter directly related to the number of clusters and \( \kappa_{RD} \) denotes the ratio of power in the LOS components to that of scattered components for R-D fading links and \( \theta_{rd} \) denotes the average SNR of the source-to-destination fading link. The error probability of the cooperation mode, \( P_{SE}^{S-D,R-R-D} \) may be expressed as [41, 44]:

\[
P_{E}^{S-D,R-R-D} = \left\{ \begin{array}{l}
\frac{a}{\pi} \int_{0}^{\pi} M_{fa} \left( \frac{b}{2 \sin^2 \theta} \right) d\theta \\
\frac{c}{\pi} \int_{0}^{\kappa} M_{fa} \left( \frac{b}{2 \sin^2 \theta} \right) d\theta
\end{array} \right\} \times
\left\{ \begin{array}{l}
\frac{a}{\pi} \int_{0}^{\pi} M_{fa} \left( \frac{b}{2 \sin^2 \theta} \right) d\theta \\
\frac{c}{\pi} \int_{0}^{\kappa} M_{fa} \left( \frac{b}{2 \sin^2 \theta} \right) d\theta
\end{array} \right\}.
\]

(18)

\( P_{E}^{S-D,R-R-D} \) represents the cooperation-based signal transmission mode. If the relay decodes correctly during the relaying phase, then the destination gets the signal from the relay node and from the source node. The optimal combination is performed at the destination node using maximal ratio combining schemes. End-to-end SER of the cooperative communication fading link is:

\[
P_{E} = P_{E}^{S-R} \times P_{E}^{S-E} + \left( 1 - P_{E}^{S-R} \right) \times P_{E}^{S-D,R-R-D}.
\]

(19)

The end-to-end SER of the cooperative communication fading link may be obtained by substituting Eqs. (8), (12) and (18) into Eq. (19).

3. Simulation Results

For a MIMO-STBC S-DF relaying network, we demonstrate simulation plots of the average SER over non-homogeneous fading channel conditions. Monte Carlo simulations are conducted, and Matlab software is used for the simulations. In Figs. 1–3, for simplicity reasons, we take \( \mu_{RD} = \mu_{SD} = \mu_{SR} = \mu \) and \( \kappa_{RD} = \kappa_{SD} = \kappa_{SR} = \kappa \). The theoretical expressions in \( \kappa-\mu \) fading are in the form of infinite series. However, these series converge very rapidly with an increase in the number of summation terms (\( N \)), e.g., \( N = 15 \) is enough to attain accuracy up to 4 decimal points. For better and assured precision, a corresponding analysis is performed with \( N = 20 \). In Fig. 1 we considered equal power.
allocation factors with Q-PSK modulated symbols. The average end-to-end error probability versus SNR plots are shown with clear detection of $\kappa$-$\mu$ over fading channels. Average SER is plotted for $\mu = 1$, and varying $\kappa$ using Eq. (16). We observe that the increment in performance is higher along with the increase in $\kappa$. In Fig. 2, SER vs. SNR is plotted for various values of $\kappa$ and for a fixed value of $\mu$. In Fig. 3, a SER vs. SNR plot is given for various values of $\mu$ and for a fixed value of $\kappa$. It has been shown that SER performance improves with the increase in the value of $\mu$.

**Fig. 1.** Average SER performance of 4-PSK over $\kappa$-$\mu$ fading channels with different values of $\kappa$.

**Fig. 2.** Average SER performance of 4-QAM over $\kappa$-$\mu$ fading channels with different values of $\kappa$.

**Fig. 3.** Average SER performance of 4-PSK over $\kappa$-$\mu$ fading channels with different values of $\mu$.

4. Conclusions

We have investigated CF expressions of the average SER for a MIMO STBC S-DF relaying network over $\kappa$-$\mu$ faded links when the input is Q-PSK and 4-QAM modulated. The average SER of QAM and QPSK are presented in the figures. Specifically, we consider the case where the $S\rightarrow R$, $R\rightarrow D$ and $S\rightarrow D$ link is subject to the i.i.d. $\kappa$-$\mu$ fading. From the simulation results, an enhancement in SER with a stronger LOS component is observed.

**Appendix A**

Solution of $I_1 = \frac{a}{\pi} \int_0^{\pi} M_{pr} \left( \frac{b}{2 \sin^2 \theta} \right) d\theta$:

Let

$$t = \frac{\mu_{sr}(1 + \kappa_{sr})}{\mu_{sr}(1 + \kappa_{sr}) + \frac{\mu_{sr} \beta_{sr}}{2 \sin^2 \theta}}.$$  \hspace{1cm} (20)
After performing some manipulations, \( \sin^2(\theta) \) is expressed as:

\[
\sin^2(\theta) = \frac{b \mathcal{T}_{1r} t}{2 \mu_{sr}(1 + \kappa_{sr})(1 - t)} .
\]  
(21)

After differentiating Eq. (21) with respect to \( dr \), we get:

\[
2 \sin(\theta) \cos(\theta) \, dr = \frac{b \mathcal{T}_{1r} t}{2 \mu_{sr}(1 + \kappa_{sr})(1 - t)^2} .
\]  
(24)

The lower and upper limits of integral \( I_1 \) changes from 0 to \( \frac{\pi}{2} \) and from \( \frac{\pi}{2} \) to \( \mu_{sr}(1 + \kappa_{sr}) + \frac{2b \mathcal{T}_{1r}}{2} \).

Substituting Eqs. (21)–(24) into integral \( I_1 \), we get:

\[
I_1 = \frac{a}{\pi} \int_{0}^{\mu_{sr}(1 + \kappa_{sr}) + \frac{2b \mathcal{T}_{1r}}{2}} \frac{b \mathcal{T}_{1r} t}{2 \mu_{sr}(1 + \kappa_{sr})(1 - t) - b \mathcal{T}_{1r} t} .
\]  
(25)

For further simplification of this integral, it may be brought in the form of a confluent hypergeometric function with the substitution:

\[
y = \frac{2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t}{2 \mu_{sr}(1 + \kappa_{sr})} .
\]  
(26)

The above substitution converts the upper limit of the integral to unity without changing the lower limit of the integral. This makes it easy to represent this integral into the standard form of a confluent hypergeometric function of two variables. The confluent hypergeometric function is defined as:

\[
\Phi_1(a, b, c, x, y) = \frac{\Gamma(c)}{\Gamma(a) \Gamma(c - a)} \int_{0}^{1} t^{a-1} (1-t)^{c-a-1}(1-xt)^{-b} e^{yt} \, dt .
\]  
(27)

Further simplifications after substitution of Eq. (26) into Eq. (27) bring the integral in the form that may be given as:

\[
I_1 = \frac{a}{\pi} \int_{0}^{\frac{1}{2} \mu_{sr}(1 + \kappa_{sr}) + \frac{2b \mathcal{T}_{1r}}{2}} \frac{b \mathcal{T}_{1r} y^\mu - \frac{2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t}{2} \left( 1 - 2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t \right)^{-\mu_{sr} + \frac{1}{2}}}{2 \sqrt{b \mathcal{T}_{1r} t} \sqrt{2 \mu_{sr}(1 + \kappa_{sr})(1 - t) - b \mathcal{T}_{1r} t}} \, dy .
\]  
(28)

Thus, \( I_1 \) may be finally evaluated in the form of a confluent hypergeometric function as:

\[
I_1 = \frac{a}{\pi} \int_{0}^{\frac{1}{2} \mu_{sr}(1 + \kappa_{sr}) + \frac{2b \mathcal{T}_{1r}}{2}} \frac{b \mathcal{T}_{1r} y^\mu - \frac{2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t}{2} \left( 1 - 2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t \right)^{-\mu_{sr} + \frac{1}{2}}}{2 \sqrt{b \mathcal{T}_{1r} t} \sqrt{2 \mu_{sr}(1 + \kappa_{sr})(1 - t) - b \mathcal{T}_{1r} t}} \, dy .
\]  
(29)

The above expression may be compared with the definition of the confluent hypergeometric function given in Eq. (27) to obtain the arguments of the function as:

\[
a = \mu_{sr} + 1 ,
\]  
(30)

\[
b = 1 ,
\]  
(31)

\[
c = \mu_{sr} + 1 ,
\]  
(32)

\[
x = \frac{2 \mu_{sr}(1 + \kappa_{sr})}{2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t} ,
\]  
(33)

\[
y = \frac{2 \mu_{sr}(1 + \kappa_{sr})}{2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t} .
\]  
(34)

Next, we will discuss the solution of \( I_2 \).

**Appendix B**

One may proceed to the solution of \( I_2 \) following the steps used for the solution of \( I_1 \) earlier. We make the same substitution as made for \( I_1 \) in Eq. (26). Thus, the same expressions will be used in the integral as discussed in Eqs. (30)–(34). However, the upper limit of the integral will now be \( \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t \).

Now, it can be represented as:

\[
I_2 = \frac{c}{\pi} \int_{0}^{\frac{1}{2} \mu_{sr}(1 + \kappa_{sr}) + \frac{2b \mathcal{T}_{1r}}{2}} \frac{b \mathcal{T}_{1r} y^\mu - \frac{2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t}{2} \left( 1 - 2 \mu_{sr}(1 + \kappa_{sr}) + b \mathcal{T}_{1r} t \right)^{-\mu_{sr} + \frac{1}{2}}}{2 \sqrt{b \mathcal{T}_{1r} t} \sqrt{2 \mu_{sr}(1 + \kappa_{sr})(1 - t) - b \mathcal{T}_{1r} t}} \, dy .
\]  
(36)
For further simplification of this integral, it may be brought in the form of a confluent Lauricella’s hypergeometric function of three variables with the following substitution:

\[ y = \frac{\mu_r (1 + \kappa_r) + b \Psi_r}{\mu_r (1 + \kappa_r)} t \, . \quad (37) \]

The above substitution converts the upper limit of the integral to unity without changing the lower limit of the integral. This makes it easy to represent this integral into the standard form of a confluent Lauricella’s hypergeometric function of three variables. It is defined as:

\[ \phi_i^{(3)}(a,b_1,b_2,c,x,y,z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)} \int_0^1 t^{a-1} (1-t)^{c-a-1} \]

\[ (1-xt)^{-b_1}(1-yt)^{-b_2}e^{zt} \, dt \, . \quad (38) \]

Further simplifications after substitution of Eq. (37) into Eq. (36) bring the integral in the form that may be expressed as:

\[ I_2 = \frac{c}{\pi} \sqrt{b \Psi_r (\mu_r \kappa_r) \mu_r} \left( \frac{\mu_r (1 + \kappa_r) + b \Psi_r}{\mu_r (1 + \kappa_r) + b \Psi_r} \right) \mu_r \frac{1}{2} \times \]

\[ \frac{1}{\sqrt{2\mu_r (1 + \kappa_r)}} \left( 1 - \frac{2\mu_r (1 + \kappa_r) + b \Psi_r}{2\mu_r (1 + \kappa_r) + 2b \Psi_r} \right)^{-\frac{1}{2}} dy \, . \quad (39) \]

The above expression may be compared with the definition of the confluent Lauricella’s hypergeometric function given in Eq. (38) to obtain the arguments of the function as:

\[ a = \mu_r + \frac{1}{2} \, , \quad (40) \]

\[ b_1 = 1 \, , \quad (41) \]

\[ b_2 = \frac{1}{2} \, , \quad (42) \]

\[ c = \mu_r + \frac{3}{2} \, , \quad (43) \]

The condition for convergence of this function is \(|x| < 1\) which is satisfied in our case for all values of average SNR.

Thus, \( I_2 \) may be finally evaluated in the form of a confluent Lauricella’s hypergeometric function as:

\[ I_2 = \frac{c}{\pi} \sqrt{b \Psi_r (\mu_r \kappa_r) \mu_r} \left( \frac{\mu_r (1 + \kappa_r) + b \Psi_r}{\mu_r (1 + \kappa_r) + b \Psi_r} \right) \mu_r \frac{1}{2} \times \]

\[ \frac{\Gamma(\mu_r + \frac{1}{2})\sqrt{\pi}}{\Gamma(\mu_r + 1)} \left( \frac{\mu_r (1 + \kappa_r) + b \Psi_r}{\mu_r (1 + \kappa_r) + b \Psi_r} \right)^{\mu_r + \frac{1}{2}} \]

\[ \frac{2\mu_r (1 + \kappa_r) + b \Psi_r}{2\mu_r (1 + \kappa_r) + 2b \Psi_r} \right)^{-\frac{1}{2}} \, . \quad (44) \]

Note that \( I_1 \) and \( I_2 \) have the form, respectively, of a confluent hypergeometric function and a confluent Lauricella’s function. These functions are not commonly available in mathematical computation software. Thus, numerical evaluation methods for finite integrals may be used. Alternatively, these functions may be numerically evaluated using their series representation. The confluent hypergeometric function in the series form may be given as:

\[ \Phi_1^{(3)}(a,b,c,x,y,z) = \sum_{j=0}^{\infty} \sum_{n=0}^{\infty} \frac{(a)_{j+n}(b)_j x^n}{(c)_{j+n} n!} \, . \quad (45) \]

Using Eq. (49) into Eq. (47), the integral \( I_2 \) can be expressed as in Eq. (50).
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