Comparison of Selected Fair-optimization Methods for Flow Maximization between Given Pairs of Nodes in Telecommunications Network

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Abstract—Dimensioning of telecommunications networks requires the allocation of the flows (bandwidth) to given traffic demands for the source-destination pairs of nodes. Unit flow allocated to the given demand is associated with revenue that may vary for different demands. Problem the decision-making basic algorithms to maximize the total revenue may lead to the solutions that are unacceptable, due to “starvation” or “locking” of some demand paths less attractive with respect to the total revenue. Therefore, the fair optimization approaches must be applied. In this paper, two fair optimization methods are analyzed: the method of ordered weighted average (OWA) and the reference point method (RPM). The study assumes that flows can be bifurcated thus realized in multiple path schemes. To implement optimization model the AMPL was used with general-purpose linear programming solvers. As an example of the data, the Polish backbone network was used.

Keywords—allocation problem, decision problems, fair-optimization, linear programming, multi-criteria, networks, ordered weighted averaging, OWA, reference point method, RPM.

1. Introduction

Many times in real life people meet with decision problems affecting on different ranks to various types of business, organizations, systems, networks or other more or less complex structures. Even in the household, everyone meets with decision problems affecting to people comfort of living, safety, etc. In each of these problems, the decision-maker is taking some specific preferences and selecting according to them. If the problem can be written in the form of linear constraints and objective function, then existing software can be used to generate solutions of specific kind of problems [1], [2]. This could be the production problems, the knapsack problems, the selection of the optimal structure of the investment portfolio, whole range of problems related to networks [2] or the problem related to planning the allocation of resources [3].

Telecommunications networks are facing increasing demand for many services. Therefore, the problem to determine how much traffic of every demand (traffic stream) should be admitted into the network and how the admitted traffic should be routed through the network so as to satisfy the requirements of high network utilization and guarantee fairness to the users, is one of the most challenging problems of current telecommunications networks design [4], [5]. The problem, usually referred to as the network-dimensioning problem, is related to planning deployment of the network resources (bandwidth, link capacity, etc.) [3]–[5]. There are two main objectives against which the decision is optimized. The first one is to maximize the profit from each unit of the transmitted load on each demand, and the second one is to guarantee some fairness to prevent blocking the paths where profit is less attractive.

Figure 1 shows the case of overlapping demands. Assuming that the capacity of arc $e_3$ is less than the capacity of both $e_1$ and $e_2$ consider the problem of the allocation of load demands $d_1$ and $d_2$. Each unit load attributed to demand is profitable. When the value of the unconsolidated profit will be different, the optimal solution may result in larger load values assigned to more profitable paths discriminating those less attractive.

![Fig. 1. An example illustrates bandwidth allocation problem on demands $d_1$ and $d_2$.](image-url)
mization have been considered and analyzed, c.f. [7], [8]. These models are:
- max-min [2],
- lexicographic max-min [2], [9], [10],
- proportional fairness (PF) [11],
- ordered weighted averages (OWA) [12], [13],
- reference point method (RPM) [14].

The most important feature in each of those methods is taking into account impartiality and equitability in the preference model. Both properties are guaranteed by comparing ordered vectors making the original sequence of objective functions not relevant for optimization. During the implementation of the methods based on ordered outcomes one can take advantage of a problem that returns the k-th largest or the smallest value of the function of resource allocation on a set of requirements \( D \) [15]. For this purpose, binary variables \( z_{kd} \) and unlimited variable \( t_k \) have been introduced. The model, which allows to receive each further objective function values in descending order can be written as follows:

\[
\begin{align*}
f(x)_k &= \min t_k, \\
\text{subject to:} \\
t_k - y_d &\geq -C_{kd}, z_{kd} \in \{0, 1\}, \forall d \in D, \\
\sum_{j=1}^{m} z_{kd} &\leq k - 1.
\end{align*}
\]

Such a formulation allows one to obtain the largest value for \( k = 1 \) from the whole set of \( f(x) \) values. Further, for any \( k > 1 \) one gets the \( k \)-th value in non-increasing order. In this case formulation brakes \( k - 1 \) restrictions using constant \( C \) with suitable large value (the largest achievement function value).

Similarly, can be determined successive values of \( f(x) \) in non-decreasing order. The corresponding optimization model can be written as:

\[
\begin{align*}
f(x)_k &= \max t_k, \\
\text{subject to:} \\
t_k - y_d &\leq C_{kd}, z_{kd} \in \{0, 1\}, \forall d \in D, \\
\sum_{j=1}^{m} z_{kd} &\leq k - 1.
\end{align*}
\]

The study is focused on OWA and the RPM models. Both methods allow to control the solution in their characteristic way, and allow to obtain results more or less fair. In determining the level of fairness, some abstract index has to be considered. It was also assumed, that in the case if for at least one of demand the allocated traffic load in the network has value of 0, then the solution is not fair. Further, fairness of solutions will become greater when the results are most aligned with each other. In statistics, this is represented by the so-called inequality measures [16]. Such measures are variance, standard deviation and kurtosis for example. Nonlinear dependencies complicate the possibility of their direct use in implementation of the large-scale network optimization model but on the other hand they can be used in the simple way to evaluate a final result of the test method.

Consider the outcome vectors set \( U \), that is the set of revenue vectors for all achievable utility allocations. The quality of obtained result when selecting the method can be assessed by the ratio defining the loss of total revenue gained from the method maximizing the revenue disregarding fairness issues. This ratio is called the price of fairness (POF) [17] and it is defined by formula:

\[
POF(U) = \frac{\max(U) - \text{fair}(U)}{\max(U)},
\]

where: \( POF(U) \) - price of fairness \( \max(U) \) - optimal solution in pure objective function maximization case \( \text{fair}(U) \) - fair solution.

2. Mathematical Models

The problem will be analyzed on sample data obtained from the library SNDlib [18]. Exactly, the topology of the Polish backbone network has been chosen. The complete graph with nodes arranged and labeled arcs is shown in Fig. 2.

![Network topology of considered optimization problem.](image)

Task of dimensioning is based on an allocation of the load on 10 pairs of source-destination demands. Pairs were selected in a way to represent a situation of overlapping paths to occur some links shared by them (Fig. 3). In order to ensure the possibility of paths bifurcation the network model is based in the node-link approach. This requires to represent the graph with an incidence matrix of vertices and arcs. Its coefficients are parameters \( a_{nl} \) and \( b_{nl} \). For each...
demand it was assigned the revenue value of the passage unit load by the demand. All of the values of variables are treated as integers. All the notations used in calculations are listed below:

- \( N \) – set of nodes (\( n_i - i \)-th node in network),
- \( L \) – set of arcs (\( l_i - i \)-th arc in network),
- \( P \) – vector of revenues per unit of capacity allocated on \( d \)-th demand (\( p_d - d \)-th demand unit revenue),
- \( D \) – set of demands between source-destination node pairs,
- \( C \) – vector of capacities of \( l \)-th arcs in the network (\( c_l - l \)-th arc’s capacity),
- \( s_d \) – source assigned to \( d \)-th demand,
- \( e_d \) – destination assigned to \( d \)-th demand.

Auxiliary parameters are:

- \( d_{il} \) – parameter having logical value which takes 1 in case of \( l \)-th arc comes out from \( n \)-th node and 0 in contrary case,
- \( b_{il} \) – parameter having logical value which takes 1 in case of \( l \)-th arc comes into from \( n \)-th node and 0 in contrary case.

Used variables:

- \( h_d \) – allocated values of bandwidth on \( d \)-th demand between source and destination point,
- \( x_{ld} \) – value of bandwidth allocated on \( l \)-th arc used in \( d \)-th demand.

Objective function:

\[
\max \sum_{d \in D} h_d p_d, \tag{4}
\]

subject to:

\[
\sum_{l \in L} d_{il} x_{ld} - \sum_{l \in L} b_{il} x_{ld} = h_d, \quad \forall d \in D, s = s_d, \tag{5}
\]

\[
\sum_{l \in L} b_{il} x_{ld} = \sum_{l \in L} d_{il} x_{ld} = h_d, \quad \forall d \in D, e = e_d, \tag{6}
\]

\[
\sum_{l \in L} b_{nl} x_{ld} - \sum_{l \in L} d_{nl} x_{ld} = 0, \quad \forall d \in D, n \in N \setminus \{s_d, e_d\}, \tag{7}
\]

\[
\sum_{d \in D} x_{ld} \leq c_l, \quad \forall l \in L. \tag{8}
\]

Equations above describe a basic node-link network model of the dimensioning problem. The main objective function (4) has been written for simple maximization problem of the total revenue. Next Eqs. (5) and (6) enforce equality of input and output throughout at each node. Limit of the maximum capacity of link is ensured by Eq. (8). Cumulative model of the ordered weighted averages allows us to formulate the OWA optimization problem in a simple form of linear programming thus guaranteeing efficient computations. The method generates the equitable efficient solutions. Control parameters of the method are weights assigned to several achievement function values, which in each subsequent step of the optimization algorithm are ordered non-decreasing. Weights must be set among the input parameters methods in non-increasing order. Their number must be equal to the number of individual functions, which in this case refers to the amount of revenue from the allocation of a specific load value on the demand. Weighting the first significantly larger than any other impact on maximizing the value of the least attractive criterion function has been achieved. Assigning equal weights values determines solution comparable to the simple maximization of the total revenue, regardless of fairness. Taking advantages of Eq. (2), the OWA model can be written as the following linear program (LP):

\[
\max \left( \sum_{i=1, \ldots, m} \eta_k \eta_k \right), \tag{9}
\]

where:

\[
\eta_k = k t_k - \sum_{i=1}^{m} d_{ik}, \tag{10}
\]

\[
t_k - d_{ik} \leq f_i(x), \tag{11}
\]

\[
d_{ik} \geq 0, \tag{12}
\]

for \( i, k = 1, \ldots, m \).

The OWA method maximizes the sum of all ordered \( k \)-th lowest objective values with assigned weights (9).

The second analyzed method (RPM) uses the fuzzy intervals. The model takes into account in its design ranges of functions achievements that are most desired by the deci-

Fig. 3. Illustration of given demands of bandwidth allocation in assumed network topology.
sion maker, defined by their limits. The aspiration and the reservation levels \((a_i; 1)\) and \(r_i\). Using the features of fuzzy intervals allows assigning function values that do not necessarily belong to the predefined interval. It is solved by introducing additional factors responsible for the decrease \((\gamma)\) “sorrow” and additional growth \((\beta)\) “satisfaction” the decision-maker with the achieved value of the function, where \(0 < \beta < 1 < \gamma\), see Eqs. (15), (16), and (17). Aspiration and reservation levels are the control parameters for this optimization method. In certain cases, these parameters can be given as the worst possible value and the best possible value, respectively. However, in most cases those control parameters are determined empirically on the basis of (or during) the problem analysis. The RPM model can be written as linear programming (LP) formulation:

\[
\max \left( z + \varepsilon \sum_{i=1}^{m} z_i \right), \tag{13}
\]

subject to:

\[
z \leq z_i, \tag{14}
\]

\[
z_i \leq \beta \frac{y_i - a_i}{a_i - r_i} + 1, \tag{15}
\]

\[
z_i \leq \gamma \frac{y_i - a_i}{a_i - r_i}, \tag{16}
\]

\[
z_i \leq \gamma \frac{y_i - a_i}{a_i - r_i}, \tag{17}
\]

for \(i = 1, \ldots, m\).

Value \(\varepsilon\), used in Eq. (13), should be positive and less than 1. It defines how important for the decision-maker is additional improvement of the total revenue. Equation (14) allows variable \(z\) to get the minimum value from \(z_i\) for each \(i = 1, \ldots, m\). This procedure uses in some part the max-min concept and guarantees the special treatment of the least attractive value in case of total system efficiency. It means how important for decision-maker is additional improvement of the solution.

### 3. Results

For implementations of the optimization models linear programming in the AMPL standard were used. The results were obtained using the GLPSOL solver and the GLPK as optimization package. The results are shown for eleven sets of income data, more precisely of given vector of revenues \(P\). For each data set has been considered several cases of different control parameters, which affect problem solution from the equitable methods. The values of profit units were generated randomly for 10 demands for allocation of load between given pairs of vertices. They are presented in Table 1. It is expected that the most discriminated demand against the method of optimizing the total profit will be those for which the gain value assigned the least and share at least one arc to another demand. For example, given the first set of input data to the unequal treatment may occur for demand \(d = 2, 7, 5\) and 10. Each of the eleven sets of input data were examined using three methods of optimization.

In the OWA method the values of weights are arranged in non-increasing order to guarantee the fairness properties. Each given weight refers to the successive value of the achievement function starting from the most “discrim-

### Table 1

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Table 4
Solutions for the first group of input parameters

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\(\bar{x}\) | 10192  | 10228  | 10115  | 12346  | 10119  | 10272  | 10160  | 10228  

As expected, methods maximizing the revenue unfairly, ignore demands, which are less attractive in case of this criterion. Those methods have returned solutions, which are unacceptable by the decision-maker taking into account the fairness criterion. Following Table 4, one can determine the values of loads of individual demands by dividing the objective functions by unit income from the given demand. Note that the values are relatively aligned with each other but the degree of fairness is fundamentally different. As mentioned, the results of Table 4 are also presented in the diagram (Fig. 4). On the horizontal axis mapped several solutions for the test methods. During the search for a solution one should be guided by the criterion of uniformity while simultaneously maximize the value of the total or average objective function. For some parameters, the OWA method returns a solution that assigns exactly the same load values as the RPM method. This is the case of situation where the value of the first weight stands out in relation to the remaining weights in the OWA method. Similar solution would be achieved when decision-maker has determined the appropriate low value of the bounds in the RPM algorithm as control parameters. These values are aiming to solve the max-min model, whose priority is to maximize the smallest value at first and then increase a total value of decision variables as much as possible. For non-zero value for the first weight and zero values for weights remaining, the OWA method returns a solution comparable to max-min solution. The OWA result in the fourth and fifth case can be considered as fair in some degree. This result is achieved for the distribution of weights where the first four are significantly larger while at the same time remaining values are lower – OWA(4). Slightly more efficient and fair solution is for linear decreasing weights – OWA(5).
The first index of quality of the analyzed methods is the price of fairness. Calculations were determined for the mean revenue value of each of eleven designing cases and described methods. Factor POF may, however, be interpreted in an ambiguous way. It does not include the value of their information on the degree of fairness of investigated method. For example, the best values of POF corresponds to MAX and OWA(3) cases which are completely unfair. It is difficult to say whether the method having the lower POF ratio is also less fair. Of course, the goal in designing efficient and fair method is to obtain the smallest possible values of POF. This index is inversely proportional to fairness and does not give any information about equity of objective functions values.

Another considered statistic is kurtosis. This measure is built on the fourth standardized moment of probability distribution and it takes a greater value, when more of the observations are placed around the mean value. This measure takes value 0 for the normal distribution. In addition, values of kurtosis close to 0 indicate the normal distribution of data sample. It follows that the decision-maker is interested in greater values of kurtosis. The analysis shows that for solutions of the pure maximizing model, kurtosis takes negative values. This affects the negative judgment in terms of fairness because it means that values of allocated loads are not uniform in high degree.

The basic statistic should also be a standard deviation which values have been calculated. This measure also determines the degree of distribution concentration, but it is less sensitive to the value of more deviating from the mean. This is because it is built on the second standardized central moment of probability distribution.

The above-mentioned basic statistics relating to the result set does not contain, however, the base fairness property. It is the assumption that the result of taking a value of 0 for at least one of the requirements should be defined as unfair. To challenge this own rate was introduced to determine the level of solutions fairness (19). It is the square root of the product of the minimum value, and the kurtosis of the sample. The higher the value, the method has higher qualities of fairness.

\[
\phi = \min_{d \in D} \sqrt{h_d \cdot K}
\]  

Table 5 presents the mean values of described statistics gained from all the considered methods and each set of given control parameters from Table 1. In summary, the analysis of two selected fairness optimization methods cannot clearly point out which method is better. The OWA method is more intuitive and does not require knowledge of the achievements of the expected objective function. For properly selected weights, the decision-maker can get a solution with varying degrees of fairness. The new-designed coefficient established to determine the degree of fairness of methods, which is averaged over 11 considered cases, speaks in favor of the reference point method. On the other hand, the POF index is clearly lower for the OWA method. Moreover, the OWA method allows the

**Fig. 4.** Comparison of fair-optimization methods for the first set of income parameters. (See color pictures online at www.nit.eu/publications/journal-jtit)

**Table 5**

<table>
<thead>
<tr>
<th>Measure</th>
<th>MAX</th>
<th>OWA(1)</th>
<th>OWA(2)</th>
<th>OWA(3)</th>
<th>OWA(4)</th>
<th>OWA(5)</th>
<th>RPM(1)</th>
<th>RPM(2)</th>
<th>RPM(3)</th>
<th>RPM(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>POF</td>
<td>0</td>
<td>0.2</td>
<td>0.19</td>
<td>0</td>
<td>0.22</td>
<td>0.18</td>
<td>0.24</td>
<td>0.28</td>
<td>0.27</td>
<td>0.23</td>
</tr>
<tr>
<td>$h_d$</td>
<td>13758</td>
<td>10906</td>
<td>11032</td>
<td>13758</td>
<td>10643</td>
<td>11202</td>
<td>9791</td>
<td>9921</td>
<td>10511</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>18730</td>
<td>9695</td>
<td>10333</td>
<td>18730</td>
<td>8580</td>
<td>8008</td>
<td>9014</td>
<td>7946</td>
<td>7606</td>
<td>9633</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0</td>
<td>20025</td>
<td>21690</td>
<td>0</td>
<td>12574</td>
<td>10854</td>
<td>23189</td>
<td>4079</td>
<td>21061</td>
<td>23343</td>
</tr>
</tbody>
</table>
decision-maker to obtain the suitable solution by assigning the control parameters – weights. In comparison to the reference point method, the OWA method can be controlled in a more friendly and intuitive way. For positive and decreasing weights, the OWA method allows to achieve result meeting high requirements for the criterion of fairness, and maximizing the load accumulated on paths. For equal weights in the OWA method, there is obtained solution identical to the simple maximization method. It can be simple used to determine the POF, which is one way to define quality of obtained solutions.

References


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