Subcarrier Gain Based Power Allocation in Multicarrier Systems

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Abstract—The Orthogonal Frequency Division Multiplexing (OFDM) transmission is the optimum version of the multicarrier transmission scheme, which has the capability to achieve high data rate. The key issue of OFDM system is the allocation of bits and power over a number of subcarriers. In this paper, a new power allocation algorithm based on subcarrier gain is proposed to maximize the bit rate. For OFDM systems, the Subcarrier Gain Based Power Allocation (SGPA) algorithm is addressed and compared with the standard Greedy Power Allocation (GPA). The authors demonstrate by analysis and simulation that the proposed algorithm reduces the computational complexity and achieves a near optimal performance in maximizing the bit rate over a number of subcarrier.

Keywords—GPA, OFDM, QAM, SNR.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a flexible and bandwidth-efficient modulation technique, which has the capability to combat Inter-Symbol-Interference (ISI) [1]. OFDM systems divide the channel into many orthogonal and tightly allocated subcarriers, which differ in signal-to-noise ratio (SNR). Resource allocation (bit and power) to the subcarriers is a fundamental aspect in the design of multicarrier system. This includes the achievement of maximum bit rate and minimum total transmit power (margin maximization) to the subcarriers and this achievement depends on the channel gain of each subcarrier. This problem has been studied in many recent researches.

The minimum power allocation for increasing the bit rate is a fundamental problem, which has a well known closed form solution called waterfilling [2], [3]. This resource allocation strategy is not practical, since the channel with best gain allocates more power than the other channel so a lot of users will not be given a chance to transmit data at all subcarriers. This problem was addressed in [4], [5], which each user is able to transmit at minimum rate and the objective was to maximize the minimum the data rate of users. In [6] a block waterfilling algorithm is proposed, which enhances the throughput (bit rate) but it gives low amount of additional computational complexity. In [7] an asymptotic Network Utility Maximization (NUM) is proposed, which maximizes network utility over the long-term throughput region by gradient-based scheduler. In [8] a novel joint bit and power allocation algorithm is proposed, which operates in fading environments and used to maximize the throughput and minimize the total transmit power, subject to a constraint on the average Bit Error Rate (BER). In [9] a dual methods based on Lagrangian relaxation are proposed, which are used to solve multiuser multicarrier resource allocation problem.

Optimal greedy power allocation already yields optimal solution for this problem [10], which can maximize the overall bit rate with the set of constraints at the expense of computational complexity [11]. Such constraints include total available power budget, number of subcarriers, target BER and QAM modulation orders. The solutions proposed in [13]–[16] are modified from the greedy algorithm, which reduce computational complexity.

In this article a new optimal Subcarrier Gain Based Power Allocation (SGPA) algorithm is introduced. This solution can decrease computational complexity and allows to achieve maximum bit rate over number of subcarriers with the same set of constraints. SGPA distributed the power among the subcarriers according to its gain. The excess (unused) power after the subcarrier gain allocation is redistributed again to reach maximum bit rate.

The rest of the paper is organized as follows. In Section 2, the standard greedy power allocation is reviewed. The proposed algorithms are presented in Section 3 and computational complexity is evaluated in Section 4. Simulation results are discussed in Section 5 and conclusions are drawn in Section 6.

2. The Greedy Algorithm

The OFDM system is characterized by an ISI channel \( H \). The \(i\)-th subcarrier can be characterized by different gains \( |H_i|, i = 1 \ldots N \), where \(N\) is number of subcarriers used to transmit number of bits equal \( b_i \) bits per symbol. The maximization of the sum bit rate can be defined by

\[
\max \sum_{i=1}^{N} b_i. \tag{1}
\]

Subjected to the constraint

\[
\sum_{i=1}^{N} P_i \leq P_{\text{budget}}, \quad \rho_{b,i} = \rho_{b,\text{target}}, \quad \text{and} \quad b_i \leq b^\text{max}, \forall i, \tag{2}
\]
where $P_i$ is the amount of power allocated to the $i$-th subcarrier, $P_{\text{budget}}$ is the total power budget, $\rho_{b,i}$ is achievable BER of $i$-th subcarrier, which assume to be equal to target BER ($P_{\text{target}}^i$) and $P_{\text{max}}$ is the maximum number of permissible bits allocated to a subcarrier. The carrier to noise ratio of the $i$-th subcarrier is given by

$$\text{CNR}_i = \frac{|H_i|^2}{N_0},$$

(3)

where $N_0$ is the total power noise at receiver. The SNR of this subcarrier is

$$\gamma_i = P_i \times \text{CNR}_i.$$  

(4)

Assume M-ary QAM modulation of order by $M_k$ and $1 \leq k \leq K$ are the QAM levels, which is given by [11]

$$M_k = 2^{b_k} \text{ for } 1 \leq k \leq K, \text{ and } M_k = 0 \text{ for } k = 0. \quad (5)$$

The BER of this QAM modulation is [12]

$$\rho_b = \frac{1 - \left[ 1 - 2 \left( 1 - \frac{1}{\sqrt{M_k}} \right) \mathbb{Q} \left( \sqrt{\frac{3\gamma_i}{M_k - 1}} \right) \right]}{\log_2 M_k},$$

(6)

which $\mathbb{Q}$ is the well-known $Q$ function and its inverse is $Q^{-1}$. To achieve a throughput $b_k = \log_2 M_k$ with BER of $\rho_{b,\text{target}}$ minimum SNR is required as

$$\gamma_k^{\text{QAM}} = \frac{M_k - 1}{3} \left[ Q^{-1} \left( 1 - \frac{1 - \rho_{b,\text{target}} \log_2 M_k}{2 \left( 1 - \frac{1}{\sqrt{M_k}} \right) \mathbb{Q} \left( \sqrt{\frac{3\gamma_i}{M_k - 1}} \right)} \right) \right],$$

(7)

To perform greedy power allocation, Uniform Power Allocation (UPA) must be performed at first, which can be summarized as follows [11], [13], [14]:

- Allocate the power budget equally among all subcarriers. The SNR of subcarriers $\gamma_i$ in Eq. (4) is given by

$$\gamma_i = P_i \times \text{CNR}_i.$$  

(8)

- Calculate SNR of QAM levels $\gamma_k^{\text{QAM}}$ for all $M_k$, $0 \leq k \leq K$ by using Eq. (7);

- Distribute subcarriers according to their SNRs $\gamma_i$ into QAM groups $G_k$, $0 \leq k \leq K$ bounded by QAM levels $\gamma_k^{\text{QAM}}$ and $\gamma_{k+1}^{\text{QAM}}$ such as

$$\gamma_i \geq \gamma_k^{\text{QAM}} \text{ and } \gamma_i \leq \gamma_{k+1}^{\text{QAM}},$$

(9)

with $\gamma_0^{\text{QAM}} = 0$ and $\gamma_{K+1}^{\text{QAM}} = \infty$;

- Load all subcarrier with QAM orders $M_k$ according to their $\gamma_i$ in Eq. (4) then compute total allocated bits to all groups

$$B_u = \sum_{i=1}^{N} b^*_i = \sum_{i=1}^{N} \log_2 M_{b^*_i},$$

(10)

where the sub-index $u$ represents UPA.

Then there would be some unused (excess) power

$$P_{\text{ex}} = \sum_{i=1}^{N} \frac{\gamma_i^{\text{QAM}}}{\text{CNR}_i} = P_{\text{budget}} - \sum_{i=1}^{N} \frac{\gamma_i^{\text{QAM}}}{\text{CNR}_i},$$

and the power used in UPA is

$$P_{\text{used}} = P_{\text{budget}} - P_{\text{ex}}.$$  

(11)

After this step GPA can be applied [11], [13]–[19], which performs an iterative redistribution of the excess power of UPA on all subcarriers. At each iteration, the Algorithm 1 tries to increase bit rate by upgrading the subcarrier of the least require power to the next higher QAM level [11], [13]. It stops when the remaining power of excess power cannot raise any subcarrier to the next level.

**Algorithm 1: GPA**

**Initialization:**

Initiate power for GPA to $P_{\text{uppa}} = P_i^u$ in (11)

For each subcarrier $i$ do the following:

1. Set $b_i^{\text{uppa}} = b_i^u$ in (10) and $k_i = k$ in (9)

2. Calculate the minimum required upgrade power:

$$P_{i}^{\text{up}} = \frac{\gamma_i^{\text{QAM}} - \gamma_{i+1}^{\text{QAM}}}{\text{CNR}_i}.$$  

(11)

**Recursion:**

While $P_{\text{uppa}} \geq \min(P_{i}^{\text{up}})$ and $\min(k_i) \leq K$, $1 \leq i \leq N$

1. $j_i = \arg \min_{1 \leq j \leq N}(P_{i}^{\text{up}})$

2. $k_j = k_j + 1$, $P_{\text{uppa}} = P_{\text{uppa}} - P_{i}^{\text{up}}$

If $k_j = 1$

$$b_j^{\text{uppa}} = \log_2 M_1,$$  

else if $k_j < K$

$$b_j^{\text{uppa}} = b_j^{\text{uppa}} + \log_2 \left( \frac{M_k}{M_{k+1}} \right),$$  

else

$$b_j^{\text{uppa}} = b_j^{\text{uppa}} + \log_2 \left( \frac{M_k}{M_{k+1}} \right),$$  

$P_j^{\text{up}} = +\infty$

end

Evaluate $B_{\text{uppa}} = \sum_{i=1}^{N} b_i^{\text{uppa}}$.

3. The Proposed Algorithm

The proposed solution is referred to as Subcarrier Gain Based Power Allocation (SGPA) algorithm. It distributes the power according to subcarrier gain such that the higher gain subcarrier allocated higher power than the lower gain subcarriers. The total excess (unused) power that remains unallocated after that is redistributed to maximize the bit rate. The subcarrier gain based allocation is performed by the following procedure:
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- Calculate the weight factor according to subcarrier gain
  \[ w_i = \frac{|H_i|}{\sum_{i=1}^{N} |H_i|} \]  
  (13)
- Calculate the power of each subcarrier, which is given by
  \[ P_i = P_{\text{budget}} \cdot w_i \]  
  (14)
- The SNR of subcarrier in Eq. (4) is
  \[ y_i = P_i \cdot \text{CNR}_i = P_{\text{budget}} \cdot w_i \cdot \text{CNR}_i \]  
  (15)
- Reside subcarriers according to their SNR \( y_i \) into QAM groups \( G_k \), then total allocated bits according to Eq. (10) is
  \[ B_w = \sum_{i=1}^{N} b_i^w = \sum_{i=1}^{N} \log_2 M_{ki} \]  
  (16)
  where sub-index \( w \) represent allocating power according to weight factor.

The unused (excess) power according to Eq. (11) is
\[ P_{\text{ex}}^w = \sum_{i=1}^{N} \frac{\gamma_i - QAM_{\gamma_i}}{\text{CNR}_i} = P_{\text{budget}} - \sum_{i=1}^{N} \frac{\gamma_i - QAM_{\gamma_i}}{\text{CNR}_i} \]  
(17)
The used power for SGP A algorithm is given by
\[ P_{\text{used}}^w = P_{\text{budget}} - P_{\text{ex}}^w \]  
(18)

### 3.1. Excess Power Redistribution Mechanisms of SGP A Algorithm

This algorithm is simpler than GPA, which can reduce the complexity. It redistributes the excess power \( P_{\text{ex}}^w \) after subcarrier gain allocation to raise each subcarrier power to reach the next higher QAM level (upgrade each subcarrier one level up). This process performed over number of iteration (cycles) and stops when the remaining power of excess power cannot raise any subcarrier one level up. This solution is shown in pseudocode as Algorithm 2 and the resulting system throughput (total allocating bits) \( B_{\text{SGPA}} \) is
\[ B_{\text{SGPA}} = \sum_{i=1}^{N} b_i^{\text{SGPA}} \]  
(19)

### 4. Complexity Evaluation

The main objective of this work is to present a new algorithm, which can significantly reduce complexity compared to GPA. The computational complexities of both GPA and SGP A algorithms are presented in Table 1. The proposed SGP A algorithm upgrade each one of N-subcarriers one level up (N level upgrade for N subcarriers) in each do-while loops. Assumed that the two algorithms GPA and SGP A will perform the same number of one level upgrade \( L_1 \) then the quantity \( L_1 \) denote the average number of iterations of one do-while loop for the GPA (one level power upgrade in each loop) and \( L_2 = \inf \left( \frac{\sum}{N} \right) \) is the average number of iterations of one loop for the SGP A. In each loop, the SGP A algorithm will perform N one level power upgrade (one level upgrade for N subcarrier) but, the GPA algorithm will perform one level power upgrade for one subcarrier. In addition, the SGP A algorithm will actually require lower average number of iteration of the loop to efficiently redistribute the excess power but, the GPA algorithm require larger average number of iteration of while loop to efficiently redistribute the excess power. The redistribution of the excess power can be performed with subcarrier ordering according to subcarrier gain such that the power upgrade process start with higher gain subcarrier first. The SGP A algorithm leads to reduce the computational complexity than GPA algorithm as shown in Table 1.

From Algorithm 2, the number of operation of the SGP A algorithm that shown in Table 1, can be calculated as:

<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initiate power for SGP A to ( P_{\text{SGPA}}^i = P_{\text{ex}}^i ) in (17)</td>
</tr>
<tr>
<td>2</td>
<td>Initiate ( b_i^{\text{SGPA}} = b_i^w ) in (16) and ( k_i = k ) in (9)</td>
</tr>
<tr>
<td>3</td>
<td>Calculate ( p_i^{\text{SGPA}} = \frac{QAM_{\gamma_i} - QAM_{\gamma_i - 1}}{\text{CNR}_i} )</td>
</tr>
<tr>
<td>4</td>
<td>While ( p_{\text{SGPA}}^i \geq \min(p_i^{\text{up}}) ) and ( \min(k_i) &lt; K, 1 \leq i \leq N )</td>
</tr>
<tr>
<td>5</td>
<td>For ( j = 1 : N )</td>
</tr>
<tr>
<td>6</td>
<td>if ( p_{\text{SGPA}}^i &gt; p_i^{\text{up}} )</td>
</tr>
<tr>
<td>7</td>
<td>( p_{\text{SGPA}}^i = p_{\text{SGPA}}^i - p_i^{\text{up}}, k_j = k_j + 1 )</td>
</tr>
<tr>
<td>8</td>
<td>end</td>
</tr>
<tr>
<td>9</td>
<td>( b_{j}^{\text{SGPA}} = \log_2 (M(k_j)) ), ( p_j^{\text{up}} = \frac{QAM_{\gamma_k} - QAM_{\gamma_k - 1}}{\text{CNR}_i} )</td>
</tr>
<tr>
<td>10</td>
<td>else if ( k_j &lt; K )</td>
</tr>
<tr>
<td>11</td>
<td>( b_{j}^{\text{SGPA}} = \log_2 (M(k_j)) ), ( p_j^{\text{up}} = +\infty )</td>
</tr>
<tr>
<td>12</td>
<td>end</td>
</tr>
</tbody>
</table>

Evaluate \( B_{\text{SGPA}} = \sum_{i=1}^{N} b_i^{\text{SGPA}} \)
with two instructions. In step (8) there is if condition which perform one operation and if it is true step (9) can be performed with two instructions. Else, steps (10) (one instruction) and step (11) (two operations) can be performed or step (12) (two operations) can be performed. So the number of operations inside do-while loop equals $2N + 7N = 9N$. At step (13), N-operations are performed. So the number of operations for the SGPA (no order) equals $(L_2(9N) + 4N + 1)$ and for the SGPA (order) equals $(L_2(9N + 1) + 4N + 1)$, which add one operation inside do-while loop because of ordering the subcarriers according to its gain.

5. Simulation Results

Assume that a 32-subcarrier OFDM system is characterized by an ISI channel $H$ where the entries of $H$ are complex Gaussian random variables with zero mean and unit-variance. Fixed QAM modulation orders of $\{2^1, 2^2, \ldots, 2^{b_{\text{max}}}\}$ where $b_{\text{max}} = 8$ bit are considered and target BER = $10^{-3}$. The overall system throughput (total allocating bits) is studied and shown in Fig. 1 which shows that the performance of GPA and SGPA (order) are identical at low SNR (0 to 8 dB) which provide the same throughput but SGPA (no order) provides slightly lower throughput. At medium SNR (15 to 35 dB) GPA provides slightly higher throughput than SGPA (order and no order) by approximately 10 bits. At high SNR (> 38 dB), both the GPA, and SGPA algorithms provide the same throughput and achieve the maximum throughput of 256 bits/symbol (32 subcarriers · 8 bits) as the SNR become greater than 45 dB.

To demonstrate efficiency of the proposed scheme, the average run time of both GPA and SGPA algorithms for a 1024 subcarrier at different SNRs values is calculated, which is illustrated in Table 2. These algorithms were run on Intel Core i3 2 GHz PC with Windows 7 and MATLAB program 7.8.0. It is clear that GPA run time significantly increase with SNR and allocated power together with the number of operation (for each while loop upgrade one subcarrier one level up). On the other hand SGPA run time slightly increase due to reduced complexity (for each while loop approximately upgrade N-subcarrier N-level up).

![Fig. 1. Throughput for 32 subcarriers OFDM system with varying SNR and $P_{\text{target}} = 10^{-3}$.](image)

Table 1

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of while loop</th>
<th>Number of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPA</td>
<td>$L_1$</td>
<td>$L_1(2N + 7) + 4N + 1$</td>
</tr>
<tr>
<td>SGPA (no order)</td>
<td>$L_2 \approx \text{int} \left( \frac{L_1}{N} \right)$</td>
<td>$L_2(9N) + 4N + 1 \approx \text{int} \left( \frac{L_1(9N)}{N} \right) + 4N + 1$</td>
</tr>
<tr>
<td>SGPA (order)</td>
<td>$L_2 \approx \text{int} \left( \frac{L_1}{N} \right)$</td>
<td>$L_2(9N + 1) + 4N + 1 \approx \text{int} \left( \frac{L_1(9N + 1)}{N} \right) + 4N + 1$</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Average run time [µs]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPA</td>
<td>2.79</td>
</tr>
<tr>
<td>SGPA (no order)</td>
<td>0.064</td>
</tr>
<tr>
<td>SGPA (order)</td>
<td>1.93</td>
</tr>
<tr>
<td>(SNR = 15)</td>
<td></td>
</tr>
<tr>
<td>(SNR = 30)</td>
<td>20.6</td>
</tr>
<tr>
<td>(SNR = 45)</td>
<td>23.6</td>
</tr>
<tr>
<td>(SNR = 15)</td>
<td>0.090</td>
</tr>
<tr>
<td>(SNR = 30)</td>
<td>0.112</td>
</tr>
<tr>
<td>(SNR = 45)</td>
<td>2.03</td>
</tr>
<tr>
<td>(SNR = 45)</td>
<td>2.41</td>
</tr>
</tbody>
</table>

Figure 2 shows the average number of times to get into the while loops to perform the process of excess power redistribution that used in SGPA algorithm compared to the standard GPA for 32 and 256 subcarriers with varying SNR. The getting into while loop stops when the remaining excess power can’t raise any subcarrier on level up. In addition, the average number of while loop for the standard GPA algorithm proportionally increases with the number of subcarriers because within the one while loop, GPA raise one subcarriers one level up. On the other hand the average number of while loop for the SGPA algorithm is approximately independent on the number of subcarriers because within the one while loop, SGPA can raise all the N-subcarriers one level up. The SGPA algorithm...
has a significantly smaller average number of while loop for different number of subcarriers compared to the standard GPA algorithm. From all, the SGPA algorithm provides optimal performance in bit rate maximization and lower computational efficiency for different SNR values than GPA algorithm.

6. Conclusions

The optimum solution of maximum bit rate with minimum power is provided by greedy algorithm. However, it requires a high computational complexity. Therefore, SGPA algorithm is proposed which provides optimal performance in bit rate maximization and considerably reduces the computational efficiency compared to GPA solution.

References


Subcarrier Gain Based Power Allocation in Multicarrier Systems

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