Abstract—This paper deals with the burst ratio parameter, which describes the burstiness of a packet loss observed in digital networks. It is one of the input parameters of E-model—the most widely used method of assessing conversational quality of telephony. The burst ratio is defined for one channel scenario so it can be calculated when the whole transmission path has been characterized by a single set of parameters. The main objective of the paper is to extend the burst ratio definition when the transmission path is defined as a tandem concatenation of transmission channels being described by their individual burst ratios. It is assumed that packet loss of a single channel is described by a 2-state Markov chain. The final result of the research is an equation describing the burst ratio parameter when the transmission path consists of multiple concatenated channels. The derived formula has been validated by extensive simulations.

Keywords—burst ratio, bursty packet loss, E-Model, Quality of Service, Voice over IP.

1. Introduction

Telecommunication transmission channels can be described by multiple parameters. One of them is packet loss, describing the probability that a packet was not correctly delivered. This degradation can be caused by multiple factors, e.g., the transmission link error, congestion or failure of the transmission device. The effect of packet loss greatly influences the quality of real-time services—if just a fraction of packets are not delivered, the video conference or phone call service can be regarded as unusable [1], [2].

In order to better understand the behavior of packet networks, packet loss models have been developed. One of the common approaches is to model packet loss using a 2-state Markov model [3], which is a special case of the Gilbert-Elliott model [4]. It describes dependency in packet loss by introducing two transmission channel states: transmitting and losing, as well as the probabilities of changing each state. Although more sophisticated models exist, like the 4-state Markov model [5] or n-state models [6], the 2-state Markov model has been proven to correctly reflect network performance if it does not include long term packet loss dependencies [7]. This research is based on the 2-state Markov model, because it provides a good balance between accuracy and simplicity.

Packet loss ratio can be easily correlated with the perceived quality of real-time applications. However, the degree of degradation varies greatly depending on a spectrum of factors. One factor is the codec used for transmission. The incorporated techniques of packet loss recovery [8] may significantly decrease the amount of lost information and therefore improve overall quality. The next factor influencing the degree of deterioration is the size of the packet—it is clear that the loss of a packet which includes one second of a conversation has a greater influence on the quality than a loss of a 20 millisecond frame. The degree of quality degradation also depends on the packet loss distribution. It has been observed that if consecutive packets are lost, the voice or video are more impaired than when the lost packets are evenly distributed within the transmission [9], [10]. Therefore, it is very important to monitor and measure the packet loss distribution. One of its quantifiers is the burst ratio factor.

Burst ratio is used to describe packet loss distribution in digital networks. Moreover, it is one of the input arguments of the widely used analytical speech quality measurement method, the ITU-T E-model [11]. Although the definition of the parameter is simple, its usage is not very convenient. The biggest drawback is the fact that in order to use it, burst ratio must be measured for the whole end-to-end connection. However, packets are usually transmitted through numerous different networks, each characterized by its own burst ratio value. The parameter definition does not show how to calculate its value in that scenario—can the burst ratio values simply be added up to result in the cumulative value? Therefore, the influence of single channel performance on the quality perceived by end users cannot be analyzed.

Much research has been carried out on packet loss analysis using the E-model [12]–[14]. However, only one piece of research dealt with the problem of channel concatenation [15] assumes that the burst ratio of separate channels needs be multiplied in order to calculate the burst ratio of the whole connection. However, in this paper we have carefully studied the matter and we have showed that this assumption is wrong.

This paper deals with the issue by thoroughly analyzing the additivity properties of the burst ratio parameter. It gives a precise answer to the question of how to calculate the cumulative burst ratio when the whole path consists of multiple independent channels. The results of the research can be helpful in designing and managing networks as well as developing voice and video applications.

This paper is structured as follows. In Section 2 we describe in detail the burst ratio. In Section 3 we develop the equations which define the burst ratio in the multiple channels environment. In Section 4 a simplified equation
which reveals the nature of burst ratio is presented. It also includes calculations of the error induced by the simplification. Section 5 is devoted to describing and presenting the results of the simulations which validate the provided equations. Finally, the conclusions are given in Section 6.

2. Burst Ratio Overview

The burst ratio parameter (denoted in the equations as BurstR) has been defined in [16]. It quantifies the packet loss burstiness. It is calculated as the ratio of the measured average length of the packet loss bursts to the average length of the bursts in the case of random loss:

\[
\text{BurstR} = \frac{\text{Average measured burst length}}{\text{Expected average burst length for random loss}},
\]

where the burst length is the number of consecutively lost packets. In the case of random loss, the expected average burst length is given as follows (packet loss probability is denoted as \(P_{pl}\)):

\[
\mu = \frac{1}{1 - \frac{P_{pl}}{100}}.
\]

Packet losses in digital networks are commonly modeled using a 2-state Markov chain. An example of the chain is depicted in Fig. 1.

![Fig. 1. 2-state Markov loss model.](image)

In this case, if the channel successfully transmits a packet, it is in the \(F\) (Found) state. If the packet is lost, the channel is in the \(L\) (Lost) state. The \(p\) and \(q\) represent the probabilities of the channel switching between the \(L\) and \(F\) states. According to [11], given the channel modeled in this way the BurstR can be calculated as:

\[
\text{BurstR} = \frac{1}{p + q},
\]

while the packet loss is:

\[
P_{pl} = \frac{p}{p + q} \cdot 100.
\]

In the next section the additivity properties of the burst ratio are investigated.

3. Burst Ratio Calculation over Concatenated Channels

This section deals with the problem of using the burst ratio when the transmission path is not a single element, but consists of multiple independent packet transmission channels of known characteristics. It is depicted in Fig. 2 by a path that consists of \(n\) channels, each characterized by its own packet loss rate and packet loss burstiness.

![Fig. 2. The problem of the burst ratio in merged channels network.](image)

When considering the packet loss of multiple merged channels, it must be remembered that its value in each channel is independent of other channels. Therefore, the packet loss of a path that consists of multiple channels can be described using the equations for percent addition. Eq. (5) presents the packet loss of a path that contains two channels, each characterized by packet loss, respectively \(P_{pl1}\) and \(P_{pl2}\):

\[
P_{pl1+2} = P_{pl1} + P_{pl2} - \frac{P_{pl1} \cdot P_{pl2}}{100}.
\]

The packet loss of a path that contains \(n\) multiple channels, each characterized by packet loss \(P_{pln}\), is described by the following formula for percent addition:

\[
P_{pln} = \left(1 - \prod \left(1 - \frac{P_{pln}}{100}\right)\right) \cdot 100
\]

Studying the burst ratio of the path consisting of multiple channels is much more complex. Referring to Fig. 2, if all merged channels are modeled with a 2-state Markov chain, then the analysis of the burst ratio of the whole path is the analysis of multiple Markov chains in a serial connection. A formula for the burst ratio of a path that contains only one channel is presented in Eq. (4). It contains probabilities of switching between the transmitting and losing state \((p\) and \(q\)) that characterize the channel. When the path contains two channels, Channel 1 and Channel 2, in order to calculate the BurstR\(_{1+2}\) value of the whole path, the probabilities \(p_{1+2}\) and \(q_{1+2}\) of the whole path must be known, as presented in the formula

\[
\text{BurstR}_{1+2} = \frac{1}{p_{1+2} + q_{1+2}}.
\]

Due to the fact that the considered path consists of two channels, the state of the total path is described by the pair: (state of Channel 1, state of Channel 2). Moreover,
the state of each channel can be either transmitting packets (Found) or losing packets (Lost). Therefore, the total path can be in one of four states, as depicted in Fig. 3. The figure also presents the probabilities of transitions between all the states. They use \( p_1 \) and \( q_1 \) as the probabilities of Channel 1, while \( p_2 \) and \( q_2 \) of Channel 2, as depicted in Fig. 1.

![Diagram](image_url)

**Fig. 3.** Markov chains for two merged channels.

However, in order to calculate \( \text{Burst} R_{1+2} \), the two channels must be considered together as a single path. In this case, the whole path is in the F state (as depicted in Fig. 1) only if both channels are in the Found-Found state (as presented in the Fig. 3). In all other situations the path is in the L state, as at least one channel is not transmitting. Therefore, the state transition probabilities of the whole path \( (p_{1+2} \text{ and } q_{1+2}) \) are

\[
p_{1+2} = 1 - (1 - p_1) \cdot (1 - p_2),
\]

\[
q_{1+2} = \frac{q_1 \cdot q_2 \cdot (p_1 + p_2 - p_1 \cdot p_2)}{(p_1 + q_1) \cdot (p_2 + q_2) - q_1 \cdot q_2}.
\]

Combining Eqs. (8)–(9) with Eq. (7) yields the final formula for the \( \text{Burst} R_{1+2} \) of two merged channels:

\[
\text{Burst} R_{1+2} = \frac{(p_1 + q_1) \cdot (p_2 + q_2) - q_1 \cdot q_2}{(p_1 + q_1) \cdot (p_2 + q_2) - (p_1 + p_2 - p_1 \cdot p_2)}.
\]

\[ (10) \]

\( \text{Burst} R_{1+2} \) can be presented as a function of \( \text{Burst} R \) and \( Ppl \) parameters of both channels by transforming it using Eqs. (3)–(4). The result is:

\[
\text{Burst} R_{1+2} = \frac{Ppl_1 + Ppl_2}{\text{Burst} R_1} + \frac{Ppl_3}{\text{Burst} R_2} - \frac{Ppl_3 \cdot Ppl_4}{100 \cdot \text{Burst} R_1 \cdot \text{Burst} R_2}.
\]

\[ (11) \]

It can be clearly seen that within Eq. (11), the numerator is equal to Eq. (5), while the denominator is similar to Eq. (5), but with \( \frac{Ppl}{\text{Burst} R} \) in place of the packet loss \( Ppl \) parameter. It must be remembered that Eq. (5), which is defined for two channels only, was extended to a more general case of \( n \) channels, as described in Eq. (6). Using this property, Eq. (11) can be extended to describe the burst ratio of an \( n \)-channel path. The resulting formula is:

\[
\text{Burst} R_{\Sigma} = \frac{1 - \prod (1 - \frac{Ppl}{100 \text{Burst} R})}{1 - \prod (1 - \frac{Ppl}{100 \text{Burst} R})},
\]

\[ (12) \]

Equation (12) is the final formula to calculate the burst ratio of a transmission path consisting of \( n \) channels and with only the parameters of separate channels known.

### 4. Equation Simplification

In the Section 3, the burst ratio equation in cases of channel concatenation was presented. Although this equation can be successfully used for accurate calculations, it does not reveal the nature of the burst ratio, its additivity characteristics. Therefore, in this section an approximation is presented. This simplified version of the equation makes it possible to perform quick estimations, without performing accurate but time- and power-consuming calculations.

The formula which authors found to be both simple and accurate in approximating the burst ratio of a whole path that consists of \( n \) channels is:

\[
\text{Burst} R_{\Sigma \text{simple}} = \sum Ppl_n / \sum \text{Burst} R_n.
\]

\[ (13) \]

It shows that the burst ratio of the total path can be regarded as a weighted harmonic mean of the \( \text{Burst} R \) of each separate channel. It needs to be noted that the value of the simplified equation of the burst ratio of the total path is always equal to or greater then the exact formula:

\[
\text{Burst} R_{\Sigma \text{simple}} \geq \text{Burst} R_{\Sigma}.
\]

\[ (14) \]

The error of the simplification is as follows:

\[
\varepsilon_{\text{Burst} R} = \text{Burst} R_{\Sigma \text{simple}} - \text{Burst} R_{\Sigma}.
\]

\[ (15) \]

If \( n \) channels are regarded, the largest error occurs if all of them are characterized by parameters of the same value \( (Ppl \text{ and } \text{Burst} R) \). In that case, Eq. (13) is transformed into the form:

\[
\text{Burst} R_{\Sigma \text{simple}} = \text{Burst} R.
\]

\[ (16) \]

In this situation, Eq. (12) can also be transformed, as follows:

\[
\text{Burst} R_{\Sigma} = \frac{1 - (1 - \frac{Ppl}{100 \text{Burst} R})^n}{1 - (1 - \frac{Ppl}{100 \text{Burst} R})^n}.
\]

\[ (17) \]

Transforming this equation, the formula of the burst ratio of a single channel is given by:

\[
\text{Burst} R = \frac{1 - (1 - \frac{Ppl}{100 \text{Burst} R})^n}{1 - (1 - \frac{Ppl}{100 \text{Burst} R})^n}.
\]

\[ (18) \]
Therefore, the error of simplification can be presented as

\[
\varepsilon_{Burst} = \frac{1 - \left(1 - \frac{P_{pl}}{100 \cdot BurstR_2}\right)^n}{1 - \left(1 - \frac{P_{pl}}{100 \cdot BurstR_2}\right)} - BurstR_2. \tag{19}
\]

The value of maximum possible relative error \(\eta_{Burst} (\varepsilon_{Burst} \text{ in relation to } BurstR_2)\) as a function of \(P_{pl}\) and \(BurstR_2\), if the total path consists of only two channels \((n = 2)\), is presented in Fig. 4.

\[
\varepsilon_{Burst} = \lim_{n \to \infty} \frac{1 - \left(1 - \frac{P_{pl}}{100 \cdot BurstR_2}\right)^n}{1 - \left(1 - \frac{P_{pl}}{100 \cdot BurstR_2}\right)} - BurstR_2. \tag{20}
\]

Using the properties of the natural logarithm, this equation can be transformed into the form:

\[
\varepsilon_{Burst} = \frac{\ln\left(1 - \frac{P_{pl}}{100 \cdot BurstR_2}\right)}{\ln\left(1 - \frac{P_{pl}}{100 \cdot BurstR_2}\right)} - BurstR_2. \tag{21}
\]

Figure 5 presents the relative error \(\eta_{Burst} (\varepsilon_{Burst} \text{ in relation to } BurstR_2)\) in the function of \(P_{pl}\) and \(BurstR_2\), if the path consist of \(n \to \infty\) channels, each of the same packet loss and burst ratio. This graph shows that the simplification can still be used, even if the number of concatenated channels reaches infinity, as long as the packet loss or burst ratio of the total path is small enough. In order to keep the relative error \(\eta_{Burst}\) under 5%, the packet loss of the total path needs to be under 10.5% (for \(BurstR_2 = 8\)) or \(BurstR_2\) must be under 1.7 (for \(P_{pl} = 20\%\)).

The presented results show that the burst ratio of a transmission path which consists of multiple concatenated channels can be successfully calculated using simplified Eq. (13).

5. Result Validation

In this section we present the validation methodology and validation results used to prove that Eq. (12) is the correct equation to calculate the burst ratio of a transmission path which is defined as concatenation of independent channels of known parameters.

In order to check the accuracy of the equation, simulations in Matlab were performed. The authors tried to reproduce the situation, in which data packets are transmitted through a series of channels. Each channel loses packets at a specific rate and burst ratio. Moreover, the loss is modeled using a 2-state Markov chain. The burst ratio of the total transmission path calculated using the definition Eq. (1) was compared with the value computed using Eq. (12). The latter could be calculated because the parameters of each separate transmission channel were also measured. Figure 6 presents the concept used in simulations. The simulation algorithm was designed as follows.

1. Each channel of the transmission path is modeled with a 2-state Markov chain. The \(p\) and \(q\) parameters of each Markov chain are calculated from randomly generated values of packet loss and burst ratio.
2. The first node in the chain is fed with string of zeros (0), which represent transmitted packets.
3. The input string is processed by the Markov chain. As the result, some packets are lost, which is symbolized in the string by ones (1). The Markov chain can change its state only if it is processing a non-lost packet (zero).
4. At the output of the node the packet loss ratio and burst ratio are calculated, following the definition Eq. (1). However, only packets which were not lost (marked as zeros) before entering the node are taken into consideration. Therefore, only the burst ratio and packet loss introduced by the current node are calculated.

5. The next node of the simulated transmission path is fed with the series of ones and zeros, generated in the previous step.

6. Steps 3–5 are repeated for every simulated node.

7. When all the packets are transferred through all the nodes, the burst ratio and packet loss of the whole transmission path are calculated. Here, unlike in the previous steps, the parameters are calculated using the whole output string of ones and zeros.

8. The burst ratio of the whole transmission path is also calculated using Eq. (12). Its arguments are values of burst ratio and packet loss ratio calculated in the output of each node, as described in step 4.

9. The values of burst ratio calculated in the previous two steps are compared with each other. If they are the same, this indicates that Eq. (12) is perfectly accurate.

Simulations with 4 different settings, as described in Table 1 were performed. Figure 7 includes all the results, presented as the distribution of the relative error of the burst ratio calculation performed using Eq. (12).

In the next simulation, although the number of packets used have decreased, the results are still valid, as presented in Fig. 7b. However, the smaller number of packets transmitted decreased the accuracy – in over 10% of the cases the error exceeded 3%.

Simulation 3 was performed to check if the number of channels that the transmission path consists of, influences the accuracy of the equation. The results of that simulation are presented in Fig. 7c. It can be seen that the decreased number of channels slightly improved the accuracy of the equation.

The last simulation was performed to check if the packet loss rate of the channels influence the accuracy of the equation. The results are presented in Fig. 7d. Using Eq. (12) with channels which are characterized by higher and more spread values of packet loss ratio does not have any effect on the accuracy of the calculations.

The simulations undoubtedly proved that Eq. (12) correctly calculates the burst ratio value of a transmission path which consists of multiple independent channels. However, they also showed that the equation is never 100% accurate. This is caused by the fact that the burst ratio is a function of an average length of the burst of lost packets. When considering a channel concatenation, this quantity cannot be calculated with 100% accuracy, but rather with high probability. That is why all graphs present the Gaussian function.

The simulations also showed that the number of channels and their packet loss have little or no effect on the accuracy of the results. On the other hand, the number of packets transferred has a significant influence on the precision of the burst ratio calculation: for 1 million packets the 0.95 confidence interval is at the relative error of 0.4%, while for 10 thousand packets it is 3.8%. This is why it is recom-

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**Fig. 6.** The idea used in simulations run to validate Eq. (12).

**Table 1**

<table>
<thead>
<tr>
<th>Simulation settings</th>
<th>Simulation 1</th>
<th>Simulation 2</th>
<th>Simulation 3</th>
<th>Simulation 4</th>
</tr>
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<td>Number of packets transferred</td>
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<td>10000</td>
<td>1000000</td>
<td>1000000</td>
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<tr>
<td>Number of concatenated channels</td>
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<td>10</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>Packet loss rate of each channel</td>
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<td>0–1%</td>
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<tr>
<td>Burst ratio of each channel</td>
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<td></td>
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</tr>
<tr>
<td>Number of re-runs performed</td>
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<td></td>
<td></td>
<td>1000</td>
</tr>
</tbody>
</table>
6. Conclusions

The authors have solved the problem of calculating the burst ratio when the transmission path consists of multiple channels and only the parameters of separate channels have been measured. In presented research the packet loss of each channel is modeled using a 2-state Markov chain. The solution is in the form of a function of each channel’s packet loss and burst ratio. A simplification of that function is also presented, which reveals that the burst ratio of the whole transmission path can be very well approximated with a weighted harmonic mean of separate channels’ properties. We also carried out a study of the error which is introduced by that simplification. The study provides information about the conditions under which the simplification is valid.

Moreover, the results of the simulation performed to validate the results is presented. The simulations showed that the provided results are correct regardless of the number of concatenated channels or the packet loss rate they introduce. However, they also indicated that the presented equation provides a very accurate result only when dealing with mean value of the burst ratio, rather than its instantaneous value.

The presented results can be used in QoS measurements and network performance assessment. Due to the fact that the burst ratio is mostly used as an input argument of the E-model, the study will help in assessing the voice quality in packet networks. The results can help evaluate the effect of a single transmission element on end-to-end quality. The presented simplified version of the final equation will help perform quick and simple estimations of the total burst ratio. The final formula can be used both during network modeling and monitoring, helping provide better quality in the real-time applications.

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References


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